INSTITUTE OF PHYSICS PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **25** (2004) 51–61 PII: S0143-0807(04)68116-5

How to encourage university students to solve physics problems requiring mathematical skills: the 'adventurous problem solving' approach

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Received 27 August 2003
Published 10 October 2003
Online at stacks.iop.org/EJP/25/51 (DOI: 10.1088/0143-0807/25/1/008)

Abstract

Teaching physics to first-year university students (in the USA: junior/senior level) is often hampered by their lack of skills in the underlying mathematics, and that in turn may block their understanding of the physics and their ability to solve problems. Examples are vector algebra, differential expressions and multi-dimensional integrations, and the Gauss and Ampère laws learnt in electromagnetism courses. To enhance those skills in a quick and efficient way we have developed 'Integrating Mathematics in University Physics', in which students are provided with a selection of problems (exercises) that explicitly deal with the relation between physics and mathematics.

The project is based on computer-assisted instruction (CAI), and available via the Internet[1]. Normally, in CAI a predefined student-guiding sequence for problem solving is used (systematic problem solving). For self-learning this approach was found to be far too rigid. Therefore, we developed the 'adventurous problem solving' (APS) method. In this new approach, the student has to find the solution by developing his own problem-solving strategy in an interactive way. The assessment of mathematical answers to physical questions is performed using a background link with an algebraic symbolic language interpreter. This manuscript concentrates on the subject of APS.

(Some figures in this article are in colour only in the electronic version)

[1] http://tnweb.tn.utwente.nl/onderwijs/; or http://www.utwente.nl/; search or click to: CONECT.

1. Introduction

This study addresses the question of how mathematical knowledge and skills can be integrated into studying university physics. Frequently novices not only suffer from knowledge fragmentation within physics, but also from knowledge fragmentation 'across domains': physics and mathematics are seen as independent disciplines, with no relationship between them [1, 2]. The practice of dividing introductory curricula into separate physical and mathematical courses may increase this dichotomy unless an effort is made to integrate them. Apart from introducing mathematical logic, reasoning and calculus, mathematics courses should aim to show physics students the mathematical tools that are used in physics. Such courses may introduce conceptual knowledge, but often the emphasis is on procedural knowledge, in order to promote the acquisition of mathematical skills. However, these mathematical procedures, often taught with full mathematical rigor, are hardly ever applied as such in physics. Consequently, afterwards students are still unable to apply their mathematical knowledge in a physics context [3, 4], which means that their mathematical skills are insufficient for that purpose.

An electromagnetism (EM) course is rather abstract in comparison to other first year university courses (in the USA: junior/senior level) such as mechanics. Basic EM is not really an empirical subject. Hence, instruction is more difficult, since few comparisons with the real world can be made. Due to the level of abstraction, EM requires a highly mathematical formalism. Albe *et al* [1] observed, for example, that students have problems in associating mathematical formalism with physical descriptions of the magnetic field and the flux. As a result, the lack of integration of mathematical knowledge and skills into physics is more perceptible in such a course. In conclusion, there is a need to integrate mathematical knowledge and skills into physics and support the view that mathematics is a formalism for expressing or applying conceptual understanding [3, 4].

In this study it is supposed that intentional integration of mathematics into physics courses will improve problem-solving performance because (a) mathematical techniques are related to physics concepts and therefore should be available when required and (b) a mathematical formalism is advantageous at any stage of physical problem-solving. Consequently, students should feel the need to shift from a formula-centred problem-solving strategy towards a theory-based strategy, which is grounded on concepts and principles. By 'strategy' we mean the path the student consciously plans (or should plan) to cover from the moment of posing the problem, up to the moment of presenting the answer. Mathematics should then be seen as a method to describe physics instead of being considered to be a disconnected discipline.

With this contribution we have the primary objective of introducing an efficient method to merge mathematics into physics: the adventurous problem-solving (APS) method. As a vehicle to apply APS we use the IMUP courseware (Integrating Mathematics in University Physics) [5]. Here we do not aim to present and discuss data about research concerning IMUP in full detail.

In what follows, by a 'problem' we mean an 'exercise', with or without physical context.

2. Adventurous problem solving

Normally, in CAI (computer assisted instruction) a predefined student-guiding sequence for problem solving is used (systematic problem solving approach, SPA). However, although this approach in theory may offer the most efficient problem-solving strategy, it has the disadvantage of being very rigid in the sequence of steps to be taken. Therefore, we developed the APS method. In this approach, the student has to find the solution by developing their own problem solving strategy in an interactive way.

2.1. Systematic problem-solving strategies

Mettes and Pilot [6, 7] developed a CAI model to solve physics problems in thermodynamics. van Weeren *et al* [8] simplified this model for an EM course (see figure 1 for an overview)

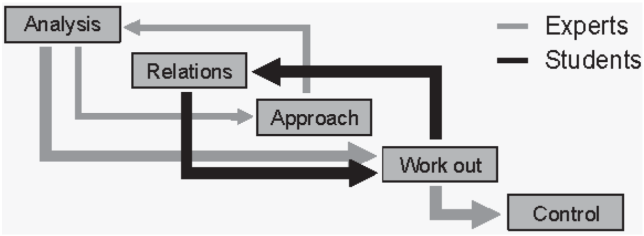


Figure 1. Typical problem-solving strategy for experts and students [8]. The 'ideal' strategy is the direct route from 'analysis' to 'control'. Typically experts concentrate on 'analysis' and 'approach', successfully followed by 'work-out', while students frequently remain trapped in 'relations' and 'work-out'-attempts.

into the main phases: 'analysis', 'relations', 'approach', 'work out' and 'control'. Reading the problem statement (including the figures) and performing an initial analysis is supposed to help students to develop a basic interpretation of the problem. Analysing the problem and looking for key relations is considered to enhance the development of a mental representation of the problem, and the establishment of an approach to tackle the problem. Then, the problem should be represented symbolically by using key relations and mathematical formalism. Subsequently, these mathematical equations should be worked out. Finally, the symbolic answer should be checked to see if it makes sense in the physical representation of the problem, by checking the dimensions and values of the calculated quantities and considering limit cases.

van Weeren *et al* [8] studied how students solved such problems. In addition, they compared students' performance to that of experts (experienced teachers or researchers). Figure 1 summarizes their findings, showing the students' use of formula-centred problem-solving strategies. Experts spent much more time in the 'analysis' and 'approach' phases than students did, which is in agreement with Hestenes' [9] observations. The implementation of their systematic approach for problem solving resulted in an improvement of successful participation of the students, but involved considerable guidance by the teacher.

However, for self-learning purposes, we found the SPA model to be far too rigid in its sequence of steps to be taken. By no means can all problems be treated in such a scheme. Secondly, too many students appeared to become very tired and even demotivated upon working with this imposed strategy. Moreover, the SPA model, although supposed to describe the ideal problem-solving strategy in science (i.e. from posing the problem, straight to presenting the answer), simply does not reflect the way scientists normally work. We decided to design a new model.

2.2. Introducing the 'adventurous problem-solving strategy'

We have devised the APS learner-controlled environment, aimed at improving problem-solving skills. Although APS might stand alone as a method, we embedded it in the IMUP courseware [5] (see below), meant to enhance the integration of mathematical and physics concepts [10], especially for the EM course. Therefore, the focus was not on knowledge alone, which is supposed not to have a positive effect in the learning process by itself [11], but on the whole problem-solving process: the representation of the physical reality, the transformation into mathematical representation, and the physical interpretation of the results. We included computer algebra calculations and added immediate feedback and guideline or help options.

The main reason to devise APS + IMUP in CAI form is that we want to encourage the students to master this subject autonomously, without the need for teachers to interfere, so that in class the teacher can concentrate on physics only, and not have to bother about mathematical skills. On the other hand, the teacher wants to obtain information about the progress of his students, and therefore we have also devised an automated analysis program.

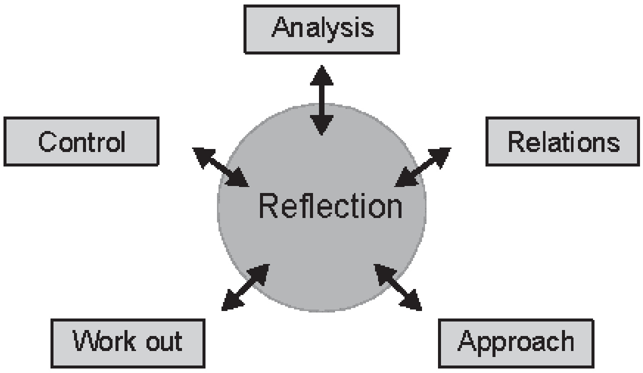


Figure 2. APS approach. Students have to find their own way through the problem, and may enter the various problem-solving phases at will, having options for reflections about how to proceed. They are free to skip phases and go directly to (e.g.) 'work-out' or 'control', when they feel having already sufficient knowledge to do so.

In the APS framework, problem solving is built in a modular way. First we divided the solution process into phases, basically following van Weeren's [8] model. Although the phases might be predetermined, in the APS approach the sequence in which the problem has to be solved is not. Students are free to complete the problem following different paths through the set of phases, seen as (virtual) 'rooms' to pass, with a special room ('general menu' room) where they can reflect on the chosen sequence and the results obtained 'so far' (see figure 2). This approach enables the students to control their progress by offering them 'intelligent' feedback upon their performance, so that the student may receive information about his progress.

The sequences that students follow to solve problems may provide information about their strategies and the development of their problem-solving skills. Therefore, students' performance was studied and compared to those of experts in order to answer the following questions:

(a) Do students perform similarly to experts in the APS framework?
(b) In which phases do they spend most of their time? and
(c) Which approach do they follow to solve the problem?

In addition, students were surveyed on their appreciation of the APS framework.

Larkin *et al* [12] observed that experts usually try to work in a straightforward direction through the phases. van Weeren [8] found that experts typically focus initially on the analysis and the approach. They may also skip phases, as a result of chunking [12, 13]. It is assumed that experts, surveying the whole problem, implicitly assess which particular problem-solving phases make sense or not in the physics context. In contrast, students use more formula-centred problem-solving strategies. Therefore, we may expect students to spend much time in sub-problems, directly trying to work out those chosen formulae. We have studied this process over time for a number of students and experts. Larkin *et al* [12] also found that experts spent less than one quarter of the time required by a novice to solve a problem. To get some insight in this point, we studied the percentage of the total time spent in each phase.

2.3. 'IMUP': Integrating mathematics in university physics

At the Faculty of Applied Physics of the University of Twente, it was observed that students did not sufficiently master the mathematical concepts and skills required for an undergraduate EM course, especially the methods that were related to integral calculus, although some dedicated mathematical courses had preceded in the curriculum (see table 1). In spite of

Table 1. Mathematical concepts and skills required for solving EM problems.

Topic	Subtopics
Vectors	Addition of vectors, modulus, unit vector, component, projection.
Coordinates	Cartesian, polar, cylindrical and spherical coordinates, transformation, symmetries, 3D insight.
Differential calculus	Differential elements, gradient, divergence, curl.
Integral calculus	Multidimensional integration, integration limits, line/surface/volume integrals; vectorial integration.

their mathematical knowledge learnt in calculus courses, students could not easily transfer that knowledge and skills to a physics context. In these calculus courses, students were first taught the mathematical concepts and methods. Then, although they extensively practised these methods, they could hardly ever relate the mathematical concepts and methods to applications in physics. As a result, students have trouble connecting the physics concepts and the mathematical concepts, which may even result in a blocking of physical knowledge because the mathematical language is not understood properly. An example is the ‘mathematical’ teaching of Gauss’ Law, which seems to offer no help when afterwards the ‘physical’ teaching is done.

A further illustration of this lack of skills is shown in the example of how to find the charge of a sphere carrying a position-dependent volume charge density. Although students from mathematics courses will probably have learned how to calculate an integral, that is not enough to know how to solve this problem. One also needs to understand what an integral stands for, in the physical and the mathematical sense. Students need to acquire concepts, like the integral being the addition of differential elements, the limits of integration corresponding to the shape of the body which is being integrated, and the description being related to the physical dimensions. Further, students need to learn skills of how to apply those concepts. Hence, students need to acquire an integrated strategy, where physical concepts and mathematical methods are related to each other.

CAI makes it possible to reproduce the physics problem-solving process step by step. CAI is a self-path instruction, which allows students to invest more time in the steps they do not understand. The CAI we developed aims at the integration of the mathematical concepts and methods in physics problem solving. Therefore, we called it the IMUP courseware.

The IMUP courseware was developed in the Dutch language, but some problems are already presented in an English translation. The courseware is based on HTML, adds intelligent behaviour by means of JavaScript, and couples with an algebraic symbolic language interpreter on the server side in order to evaluate mathematical symbolic answers to physical questions [5]. Briefly, this is done by symbolically subtracting (or dividing) the students’ answer from the correct one, and if the result yields zero (or one), the student has got the right answer. This approach enables intelligent answer checking, the use of multiple coordinate systems, intelligent dimension analysis, and evaluation of numerical answers.

3. Method and materials

3.1. The IMUP courseware

The part of the courseware that supplemented the EM course consisted of three short problems (introducing some mathematics about 3D integration) and eight complex (physical) problems (table 2). All of them belonged to the standard collection of EM problems ‘to be done’. Six complex problems were developed on the APS framework. The first two problems were developed to be completed sequentially (SPA), so that students could gradually introduce themselves into the CAI/APS framework.

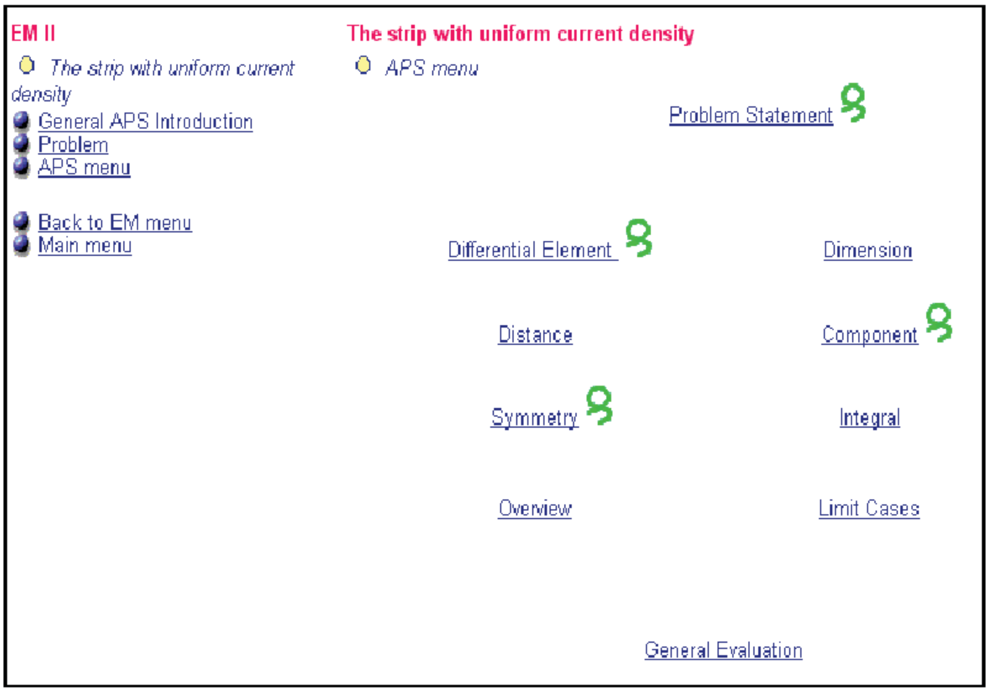


Figure 3. APS interface of problem M1 (see table 2). The student has a free choice to enter each phase indicated on the circle. However, when choosing prematurely, he will soon discover that he has to adapt his strategy. For instance, when choosing the page 'integral', he will find out that he has to complete other pages first (unless he already has sufficient background knowledge to complete the phase directly).

Table 2. IMUP exercises for APS. SPA: systematic problem-solving approach; APS: adventurous problem solving. In all problems the field vector has to be calculated, and afterwards an extension to infinite dimensions has to be carried out. E = electric; M = magnetic.

E1	SPA	Electric field of a long straight, homogeneously charged wire.
E2	SPA	Electric field of a segment of a homogeneously charged straight wire.
E3	APS	Electric field in the point above a homogeneously charged plane (using strip or ring integration).
E4	APS	Electric field above, below and in a thick charged plane with charge density varying over thickness (using integration).
E5	APS	The same, but with Gauss' law.
M1	APS	Magnetic field of a uniform surface current flowing over a strip with finite width.
M2	APS	Magnetic field on the axis of a disc with finite radius and with non-uniform circular current density (with Biot–Savart).
M3	APS	Magnetic field of a long thick wire with non-uniform current density (with Ampère's law).

3.2. The APS learner-controlled environment

The APS framework consists of an opening page including the problem statement, a graphic representation of the problem, and a brief initial analysis in the form of some questions (e.g. about symmetries). Then, the 'General Menu' screen appears. This menu shows the (clickable) names of the different phases of the solution of the exercise (see figure 3 for an example). Basically, we used the phase names as given by van Weeren [8] but, to make the concept of phases more tractable for the students, we gave them names more closely connected

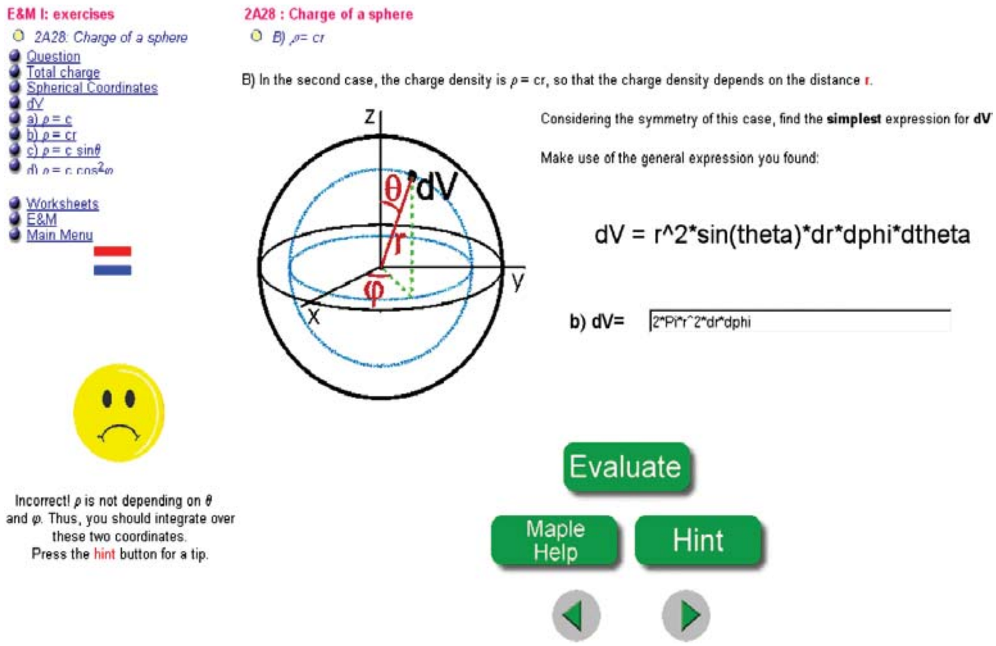


Figure 4. Typical APS page, taken from a mathematical problem supporting the EM course: '3D integration'. The student has to enter the differential form for this particular case, after having derived the form for the general case with constant charge density. When the button 'Evaluate' is pressed, the student's answer is evaluated by comparing with the proper answer using a symbolic algebra language interpreter. This procedure is applied for all symbolic-type responses the student has to enter.

to the problem at hand. The student can navigate freely from one phase to another through the APS menu. A phase that is successfully completed is marked with an approval mark. In a few cases, like the work-out (integral) phase, the answer is not evaluated at that point. In such a case, the approval mark is set only to show students that the phase was 'visited' and that an answer was given.

After the initial analysis, the first task the student should undertake is to develop an approach to solving the problem (leading to, for example, the choice between integration over rings or over strips).

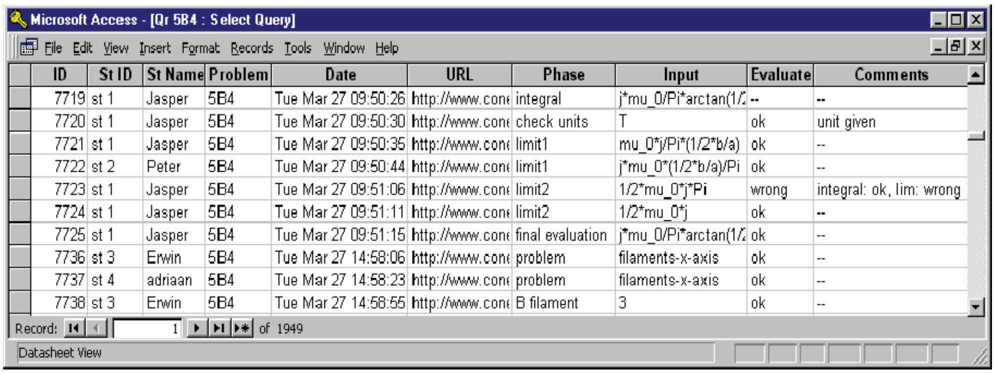
Typically the student has to enter his responses in symbolic algebraic form (figure 4), which is then evaluated by the program, using a hidden (background) link with an algebraic symbolic language interpreter, by algebraic comparison to the proper answer, and the result of this comparison is reported to the student.

The page structure is kept similar for all presented problems. The presentation of the content, as well as of the questions that are asked in each phase, is comparable from one exercise to another, although slight modifications were made to suit each problem.

The structure is flexible enough to enable the student to skip phases, if wanted, for instance in cases where the student has obtained the result of those phases (e.g. an integral expression) in another way before, such as 'using pencil and paper'.

3.3. Registration of data

Participants' performance is registered into an Access (Microsoft®) database. Each time a student enters a response to be assessed, a record (line) (figure 5) is written into the database. Each record contains information about the student and his performance:



Microsoft Access - [Qr 5B4 : Select Query]

File Edit View Insert Format Records Tools Window Help

ID	St ID	St Name	Problem	Date	URL	Phase	Input	Evaluate	Comments
7719	st 1	Jasper	5B4	Tue Mar 27 09:50:26	http://www.con	integral	j*mu_0/Pi*arctan(1/	--	--
7720	st 1	Jasper	5B4	Tue Mar 27 09:50:30	http://www.con	check units	T	ok	unit given
7721	st 1	Jasper	5B4	Tue Mar 27 09:50:35	http://www.con	limit1	mu_0*j/Pi*(1/2*b/a)	ok	--
7722	st 2	Peter	5B4	Tue Mar 27 09:50:44	http://www.con	limit1	j*mu_0*(1/2*b/a)/Pi	ok	--
7723	st 1	Jasper	5B4	Tue Mar 27 09:51:06	http://www.con	limit2	1/2*mu_0*j*Pi	wrong	integral: ok, lim: wrong
7724	st 1	Jasper	5B4	Tue Mar 27 09:51:11	http://www.con	limit2	1/2*mu_0*j	ok	--
7725	st 1	Jasper	5B4	Tue Mar 27 09:51:15	http://www.con	final evaluation	j*mu_0/Pi*arctan(1/	ok	--
7736	st 3	Erwin	5B4	Tue Mar 27 14:58:06	http://www.con	problem	filaments-x-axis	ok	--
7737	st 4	adriaan	5B4	Tue Mar 27 14:58:23	http://www.con	problem	filaments-x-axis	ok	--
7738	st 3	Erwin	5B4	Tue Mar 27 14:58:55	http://www.con	B filament	3	ok	--

Record: 1 of 1949

Datasheet View

Figure 5. Data file of the performance of the students. From these files, a student's progression through the problems can be followed and assessed, using a dedicated computer program.

(a) a record identification number (ID) uniquely identifying each entry by a student,
(b) a student identification number (St-ID),
(c) a phase column containing the particular question,
(d) an input field with the students' answer,
(e) an evaluation field where the result of the assessment of that answer is written,
(f) a comments field to register additional information on the students' performance.

3.4. Analysis of data

The analysis of the performance of the students is based on the detailed breakdown of the problems at hand into phases and sub-phases. The *phases* correspond to major steps in the problem-solving process (e.g. situation analysis, symmetry, work-out and evaluation) and the *sub-phases* are subdivisions of the phases (e.g. separate pages on screen, or different mathematical or physical steps).

In addition to the statistical features offered by the Access program we developed a specialized analysis program, written in Delphi (APS_matrix, available from the authors). This program offers the following options:

- tracking individual students (or groups of students) on their paths through the problem;
- calculating statistics of groups of students while performing a task of solving a problem;
- distinguishing between intermediate answers being 'correct' or 'wrong', while leaving a phase or a sub-phase in the problem;
- distinguishing between final results being 'correct' or 'wrong';
- assessing how the student controls the problem and performs the evaluation of his solving process;
- recording residence times spent in the (sub)-phases;
- calculating averaged residence frequencies and times, with standard deviations, and presenting those in the form of tables, or of graphical representation indicating the 'flow' through the problem.

In figure 6 a typical output of the analysis program is shown. In this case, the solving process by a group of students is analysed and averaged residence times and percentages are calculated (not shown). Also, the way the students moved from one phase to another (going in forward or backward directions) is indicated, using bars connecting the phases. The thickness of the bars indicates the frequency of occurrence, and the figure should be read in a clockwise direction.

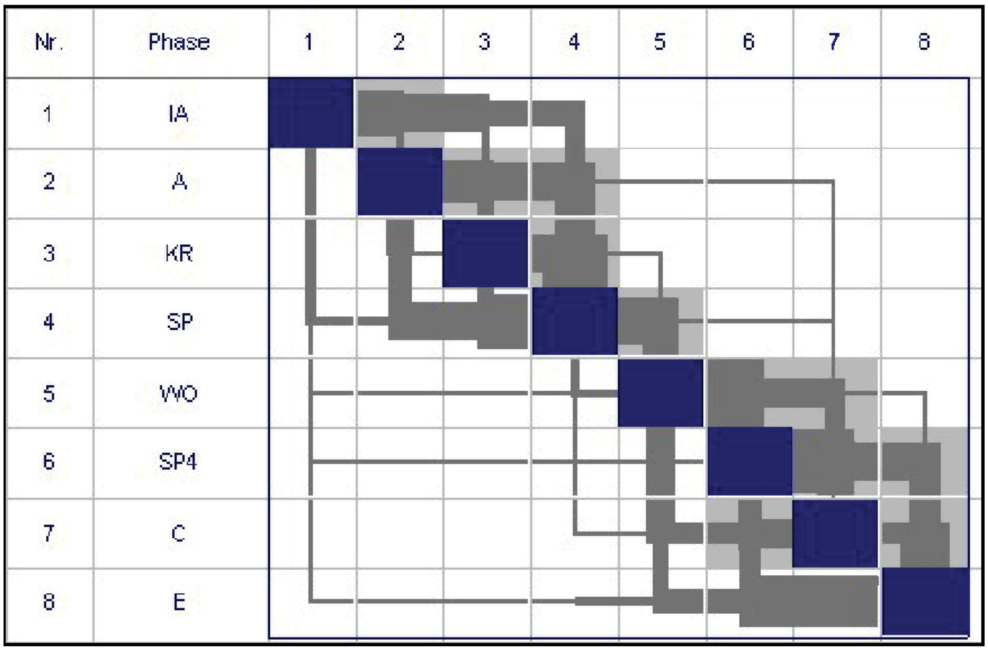


Figure 6. Output of program APS_matrix (no numerical statistics shown): students' transitions between the main phases of problem E2: IA = intro-analysis; A = analysis; KR = key relations; SP1–SP4 = physical steps; WO = work out; C = control; E = end result. Most main phases are divided into sub-phases. The transitions should be read clockwise. The thickness of the lines relates to the frequency of the transition. In light grey, possible transitions according to the ideal strategic approach (as indicated in figure 1) are shown. The statistical data (not shown) contain information about times spent in various phases and percentages for 'right' versus 'wrong' answers. The calculations can be performed for individual students or for pre/defined groups of students.

4. Results and discussion

Here we focus discussion on the method itself, and only present some typical results of the analysis. A detailed description of the results was published elsewhere [16].

Using the program APS_matrix, we analysed the process of problem solving as performed by the students and by experts. There were remarkable differences in strategy, either between students and experts on the one hand, or between students at the beginning and at the end of the course. As an example: in problem E3 (with infinite dimensions) one may choose between two ways: one by integration over strips of finite width, and another by integration over rings. A majority of students (80% of about 40 students) chose the first one, but most experts chose the second one (3 of 4).

Although the time spent in the various (sub)-phases by the students was larger in the absolute sense (by about a factor of 2), the relative time spent over the phases did not differ significantly between students and experts. This indicates that the problems did not contain questions that were extra difficult, especially for the students. The participants spent most of their time in phases where they have to give a symbolic answer (see figure 6: SP2, SP4 and WO). In such phases, the answer cannot be guessed. The fact that the program requires a specific algebraic notation may also influence the time they need to enter the response. The other phases contain either multiple-choice questions or only the evaluation of an answer to be entered.

The strategy that the experts use resembles more closely the 'ideal' problem-solving strategy than the one the students use. In addition, the experts use more forward strategies than

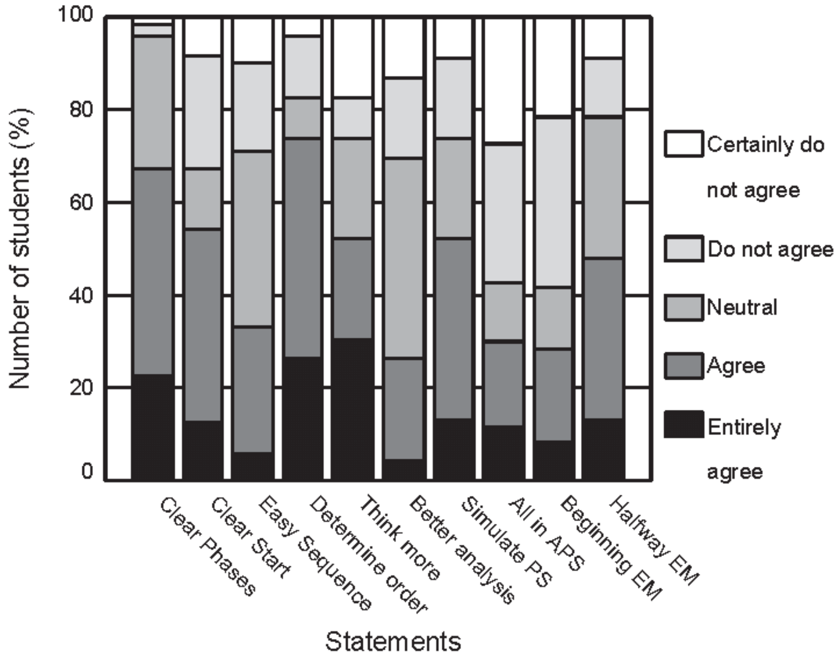


Figure 7. Frequencies (%) of 70 students agreeing with statements about APS in the questionnaire. The questions were: (1) Are the phases clearly defined? (2) Was it easy to find out how to start? (3) Was it easy to discover the proper sequence of phases? (4) Did you succeed in finding the way to solution? (5) Did APS require you to think more about the process? (6) Did APS encourage you to make a better analysis? (7) Did APS better simulate the solving process than a sequential scheme? (8) Would you like all exercises (problems) in APS format? (9) Would you like to have APS to start from the beginning of the course, or (10) from about halfway in the course?.

students, who instead use more backward strategies. The difference is significant (77% and 62% respectively, with SD $\approx$ 10). This agrees with the forward-chaining strategy observed by Larkin [14]. de Jong [15] described how students made 50% of the transitions according to that strategic model. In the present study this percentage is almost 70%. Though these two experiments are difficult to compare, it indicates that the APS environment induces students to follow a more strategic approach.

We investigated the students' opinions about the APS approach using a detailed questionnaire [16], see figure 7 for an overview. In addition, the opinions on the statements of weak and good students were compared (final marks on the EM course lower or higher than 6 out of 10, respectively). On the points of clarity and easy sequencing the first group significantly favours the SPA approach (average 3.0 ± 0.9 on a 0–5 scale), while the second group is more in favour of APS (average 4.2 ± 0.7). The results on the questionnaire show that the use of the APS framework has advantages and disadvantages. It seems to enhance problem-solving strategy, since students indicate that, using it, they learn a strategy for problem solving. In addition, they feel they can determine their own approach, which is in agreement with the aim of the APS approach. However, this approach may appear as being disorderly, especially for weak students at the start of the course, who have difficulties in finding where to start and in determining the sequence in which the problem should be solved. The results indicate that there are two categories: (a) students willing to discover the solution approach by themselves and (b) students preferring to get guidance on how to solve problems. In both cases, students think that the learning effect is in the strategic approach for problem solving.

This method of analysis could probably also be used as an attempt to quantify the 'problem-solving skills' of a student. Referring to figure 6, a sequence that follows the diagonal line from top to bottom more closely and more monotonically (less backward loops to 'previous' phases), might indicate better (i.e. more efficient and successful) problem-solving skills. This might be expressed in a 'figure of merit', inversely proportional to the number of excursions off the diagonal path and to the number of backward loops.

5. Conclusion

We have described a way to help university physics students incorporate mathematical skills into the solution process of physics problems. We devised APS, a CAI framework, in which students need to find their own way to tackle the problems. This framework was incorporated into IMUP, which has a link to an algebraic symbolic language interpreter, so that mathematical responses to physical questions, in the form of expressions entered by the student, can be assessed in a symbolical way. To date, we have devised eight electromagnetic problems (exercises) in this form.

We also developed a computerized way to analyse the progress of a student's passage through the solution process. This enables us to assess improvements in the ability of the students to tackle physics problems requiring mathematical skills, in a semi-quantitative way.

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