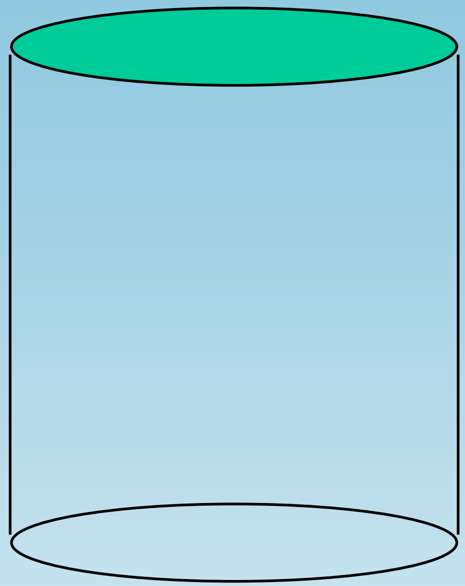
Gauss’ Law for Cylinder Symmetry

Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

Gauss' Law for Cylinder Symmetry



Available:

Cilinder, radius R, infinitely long, carrying charge density λ [C/m]

Question:

Calculate ***E***-field in arbitrary points inside and outside cilinder

Two cases:

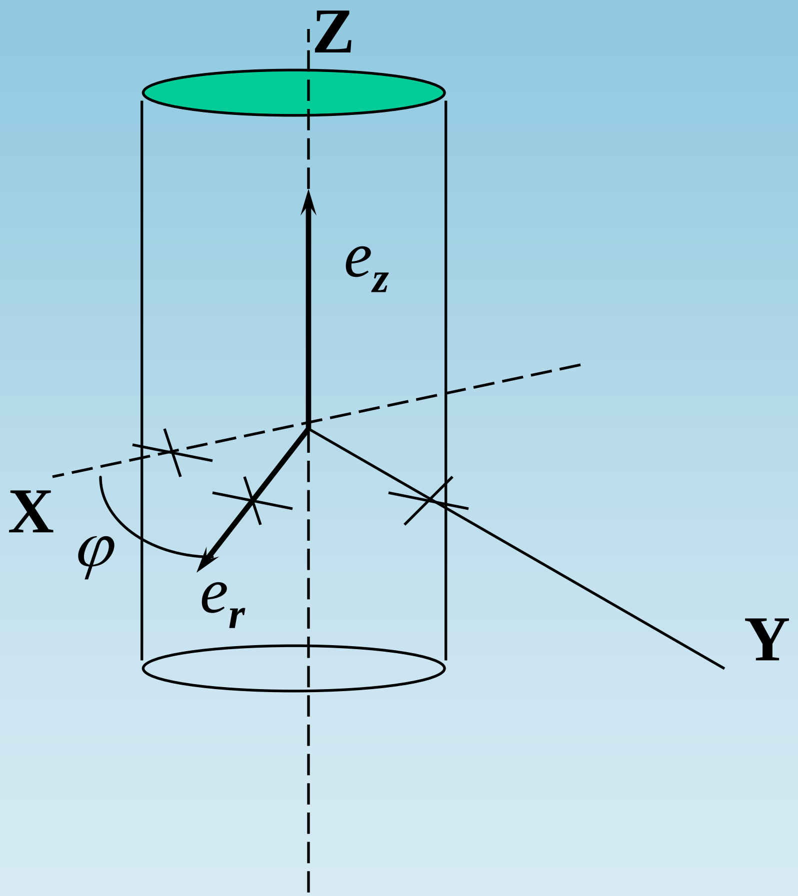
A: homogeneously charged

B: charged at surface walls only

Gauss’ Law for Cylinder Symmetry

- Analysis and symmetry
- Approach to solution
- Calculations
- Conclusions

Analysis and Symmetry (1)



1. Cylinder: infinitely long, radius R

2. Charge distribution:

   λ [C/m] ; homogeneous.

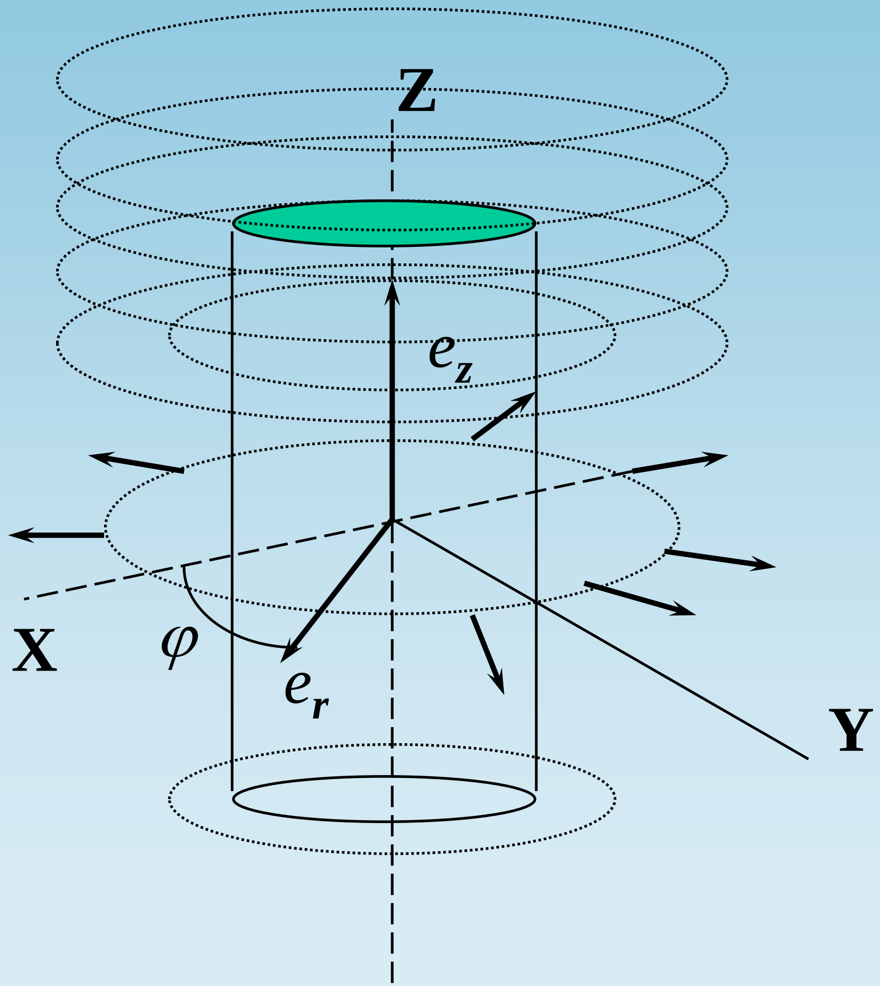
3. Coordinate axes:

   Z-axis = symm. axis

4. Cylinder symmetry:

   all points at equal r are equivalent, even if at different z or φ

Analysis and Symmetry (2)



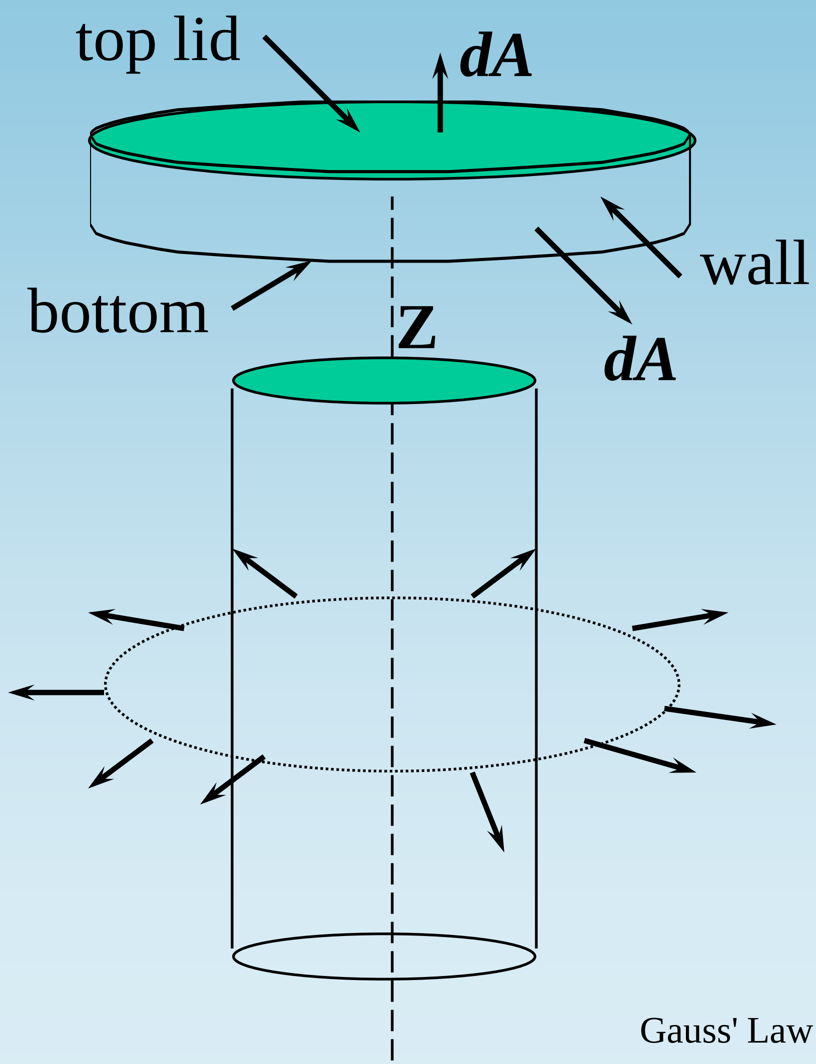
4. Cylinder symmetry:
   all points at equal r are equivalent, even if at different z or φ.

5. Consequences:
   a point charge will not move tangentially.

   E directed radially everywhere.

   all planes $z = const.$ are equivalent.

Approach to solution



Gauss' Law: $$\iint_A \boldsymbol{E} \bullet \boldsymbol{dA} = \frac{Q}{\varepsilon_0}$$

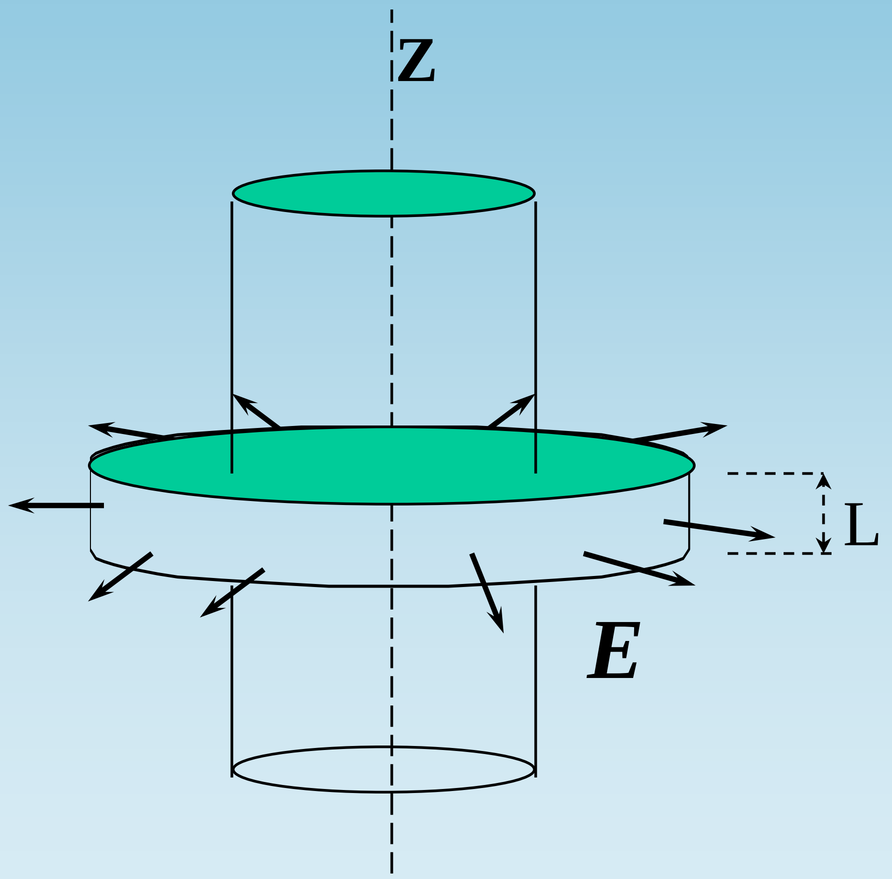
Choose Gauss-box A.

How to make optimum use of symmetry ??

- where $A \perp \boldsymbol{E}$
- where $A \,//\, \boldsymbol{E}$
- where $E = 0$??

closed box needed !!
==> pill box

Calculations (1)



Gauss' Law: $$\iint_A \boldsymbol{E} \bullet \boldsymbol{dA} = \frac{Q}{\varepsilon_0}$$

pill box: radius $r>R$; height L

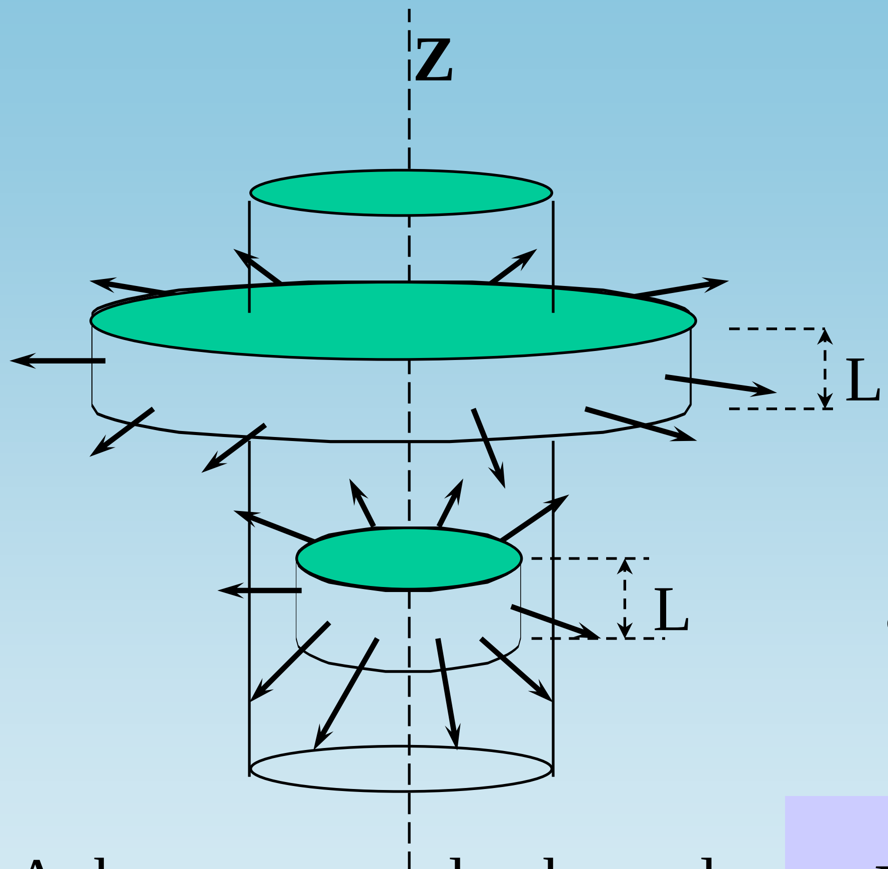
top and bottom lids do not contribute ($\boldsymbol{E} \perp \boldsymbol{dA}$)

wall contributes: $E.2\pi rL$

charge enclosed: $\lambda\ L$

result: $E(r) = \lambda\ /\ (2\pi\varepsilon_0 r)$

Calculations (2)



A: homogeneously charged
B: charged at surface only

Gauss' Law: $$\iint_A \boldsymbol{E} \bullet \boldsymbol{dA} = \frac{Q}{\varepsilon_0}$$

result: $E(r) = \lambda / (2\pi\varepsilon_0 r)$

but wait !! this holds for $r>R$!

for $r<R$: $$\iint \Rightarrow E.2\pi rL$$

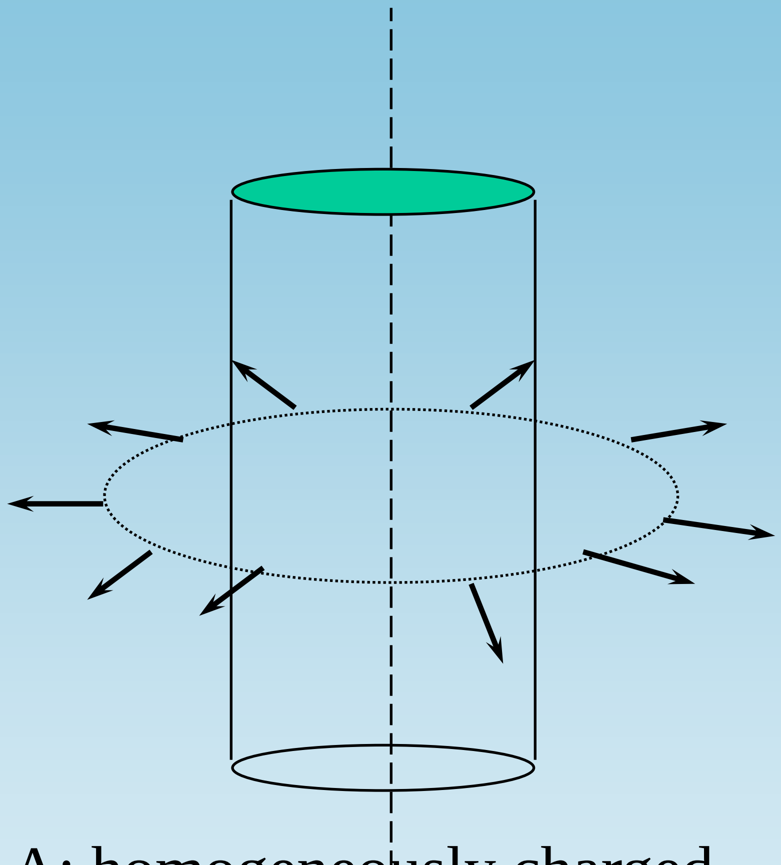
and $$Q = Q(r) = \lambda L \frac{\pi r^2}{\pi R^2} \quad \text{(case A)}$$

$$Q = 0 \quad \text{(case B)}$$

$$\Rightarrow E(r) = \frac{\lambda L}{2\pi\varepsilon_0 rL}\frac{r^2}{R^2} = \frac{\lambda r}{2\pi\varepsilon_0 R^2} \quad \text{(case A)}$$

$$E(r) = 0 \quad \text{(case B)}$$

Conclusions (1)



A: homogeneously charged
B: charged at surface only

for infinite cylinder:

$$r > R: \; \boldsymbol{E}(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\boldsymbol{e}_r$$

$$r < R: \; \boldsymbol{E}(r) = \frac{\lambda\, r}{2\pi\varepsilon_0 R^2}\boldsymbol{e}_r \quad \text{(case A)}$$

$$\boldsymbol{E}(r) = 0 \quad \text{(case B)}$$

field strength dependent of distance to cylinder =>

no homogeneous field

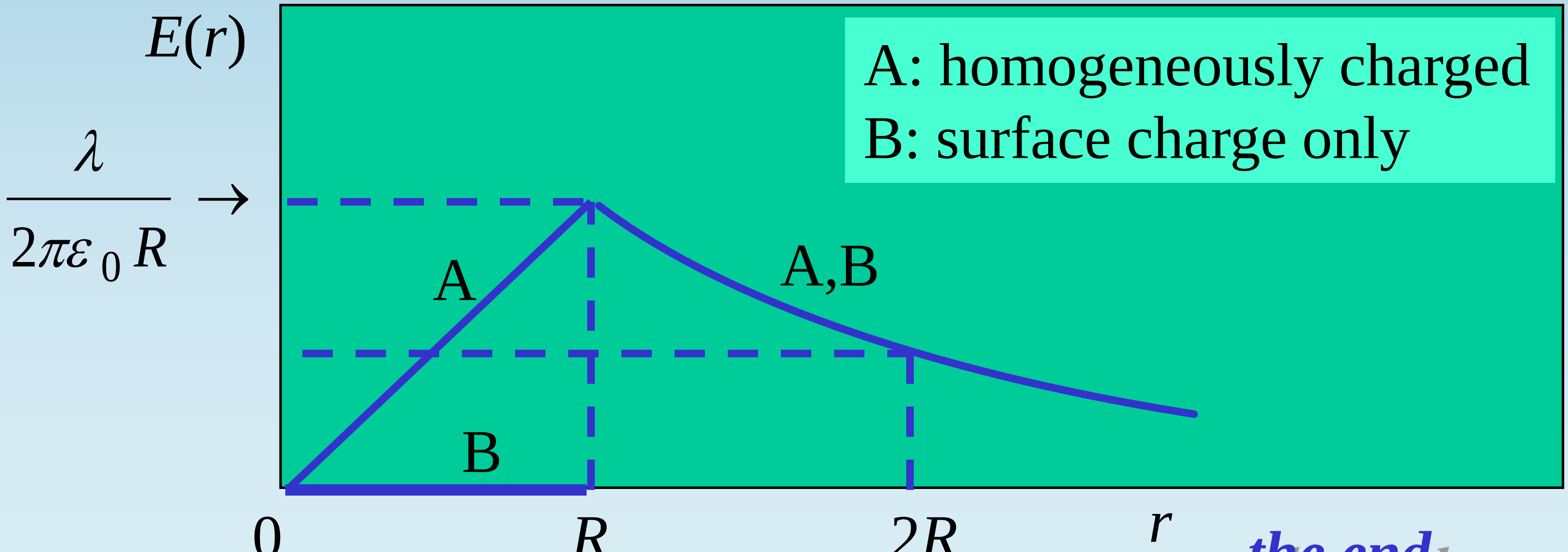
Conclusions (2)

for infinite cylinder: A: homogeneously charged;
B: surface charge only

$$r < R: \quad \boldsymbol{E}(r) = \frac{\lambda\, r}{2\pi\varepsilon_0 R^2}\boldsymbol{e}_r \quad \text{(A)}$$

$$E(r) = 0 \quad \text{(B)}$$

$$r > R: \quad \boldsymbol{E}(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\boldsymbol{e}_r$$



the end