Charge and current elements

for 1-, 2- and 3-dimensional integration

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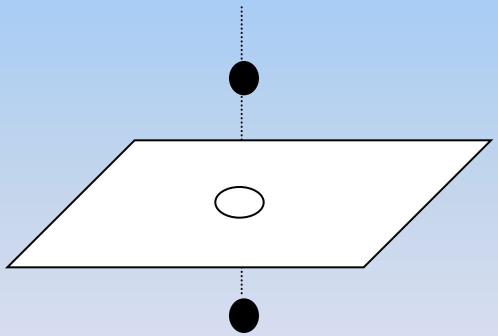
Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

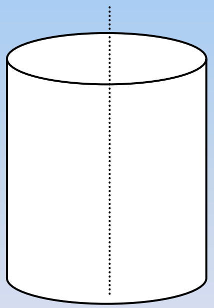
Charge and current elements for 1-, 2- and 3-dimensional integration

To perform integrations:
always rewrite **charge and current elements** *(dQ, dI)* in the form of **coordinate elements** *(dx,dy,dz* or other*)*, using **charge and current densities** (λ, σ, ρ and $\boldsymbol{j}$).

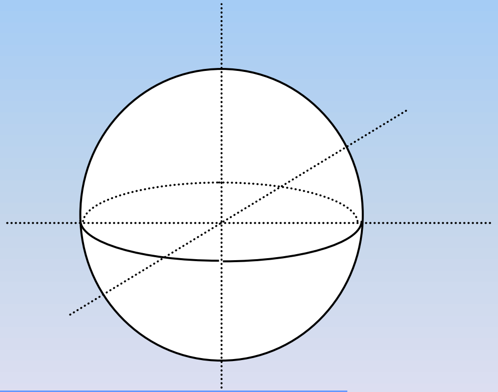
Basic symmetries



planar

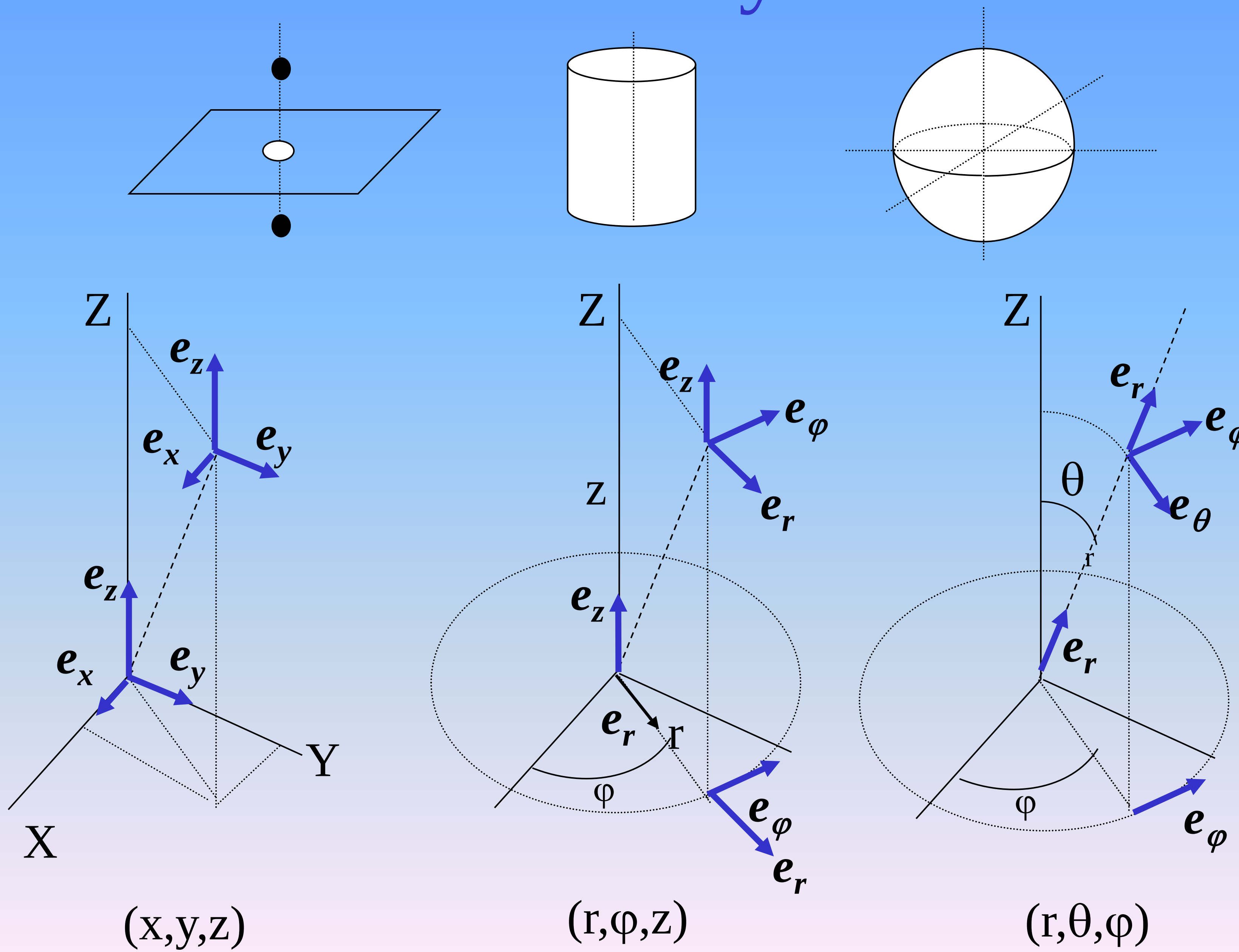


cylindrical



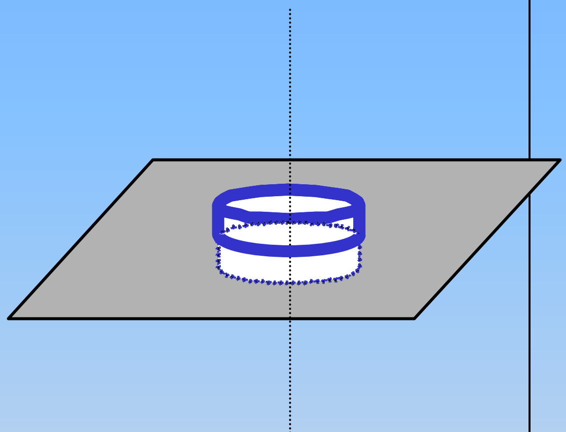
spherical

Coordinate systems



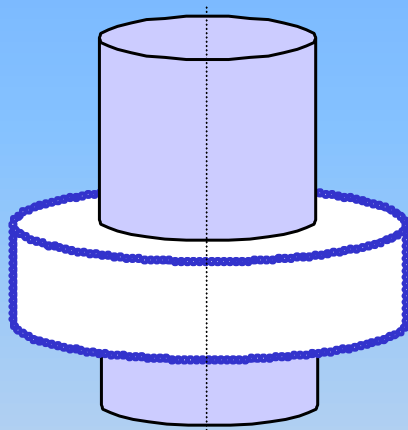
Basic symmetries for Gauss' Law

∞ extending plane



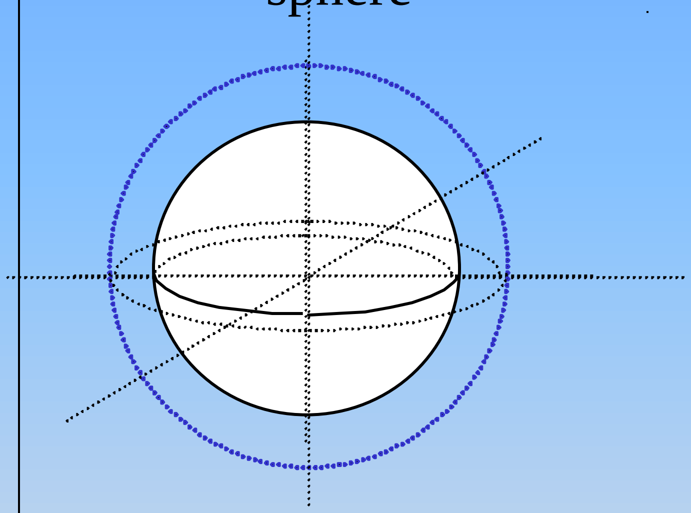
Gauss "pill box"
Height→0

∞ long cylinder



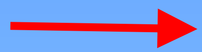
Gauss cylinder,
Radius r (r<R or r>R),
length L

sphere



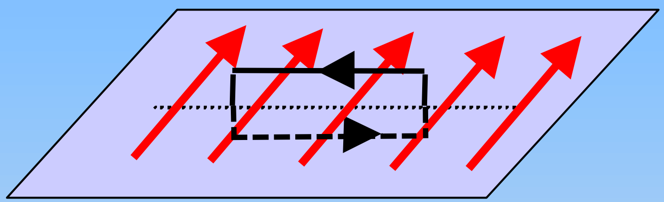
Gauss sphere,
Radius r (r<R or r>R).

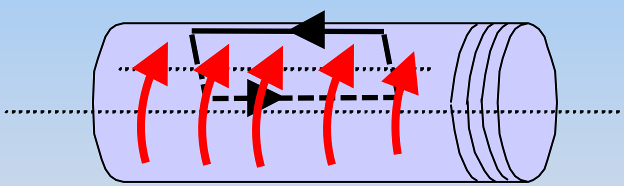
Basic symmetries for Ampere's Law


= current direction

∞ extending plane

long solenoide



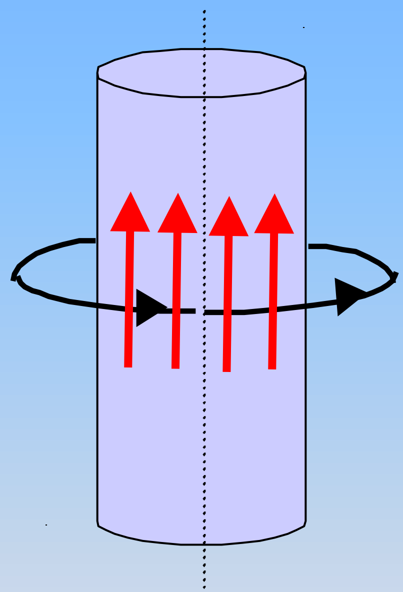


Ampère-circuit

Length L;

Height h $\rightarrow 0$

∞ long cylinder

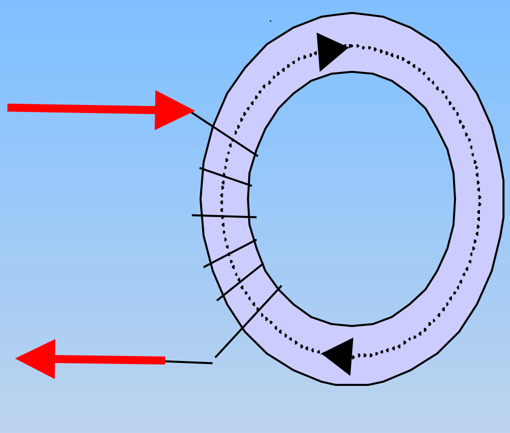


Ampère-circle,

Radius r (r<R of r>R)

toroïde

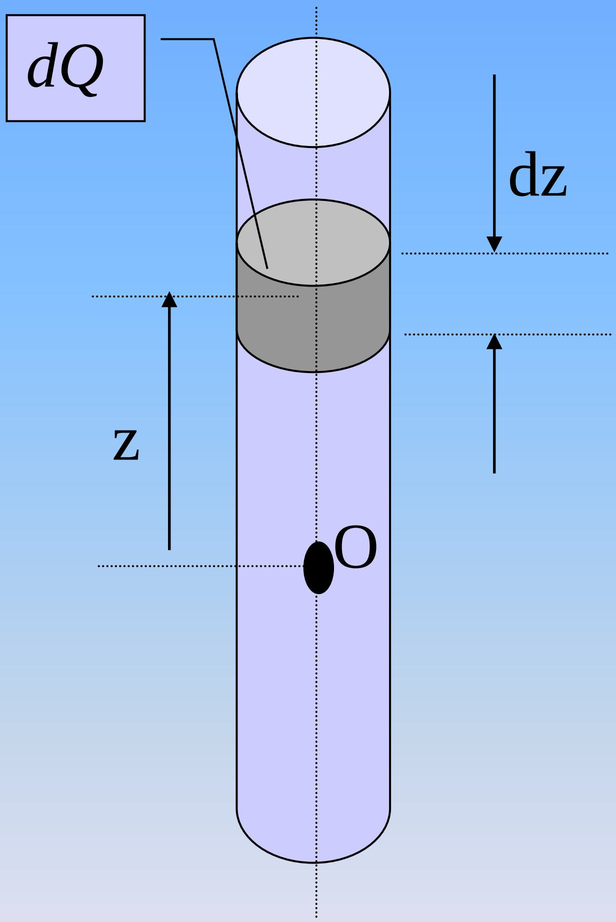
with/without gap



Ampère-circuit,

Mean circumference line , length L

Charge elements: Thin wire



Thin wire

Charge distribution: $\lambda(z)$ [C/m]

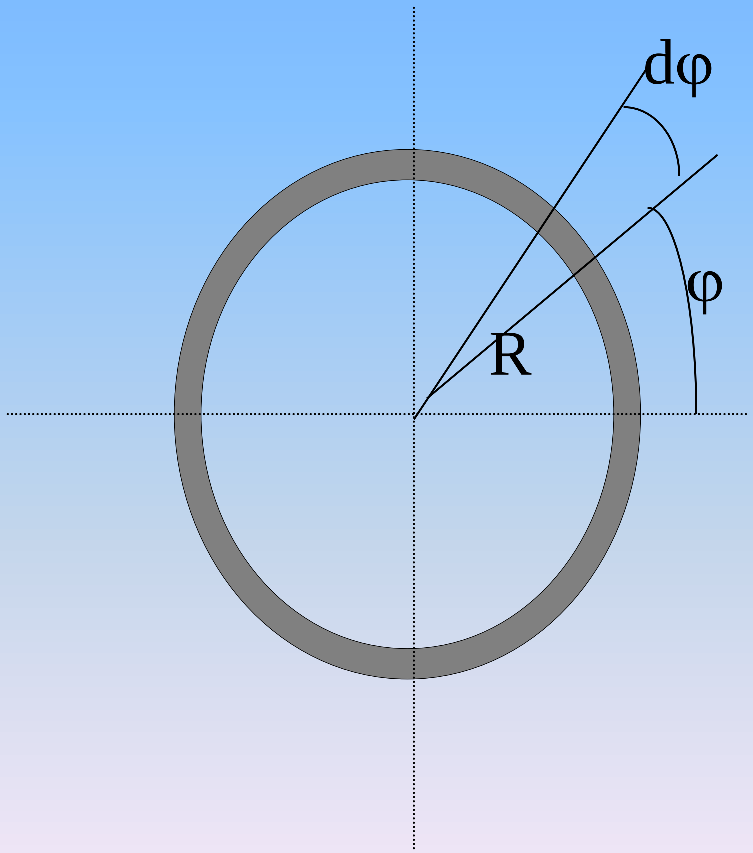
If homogeneous:

λ = const

$$dQ = \lambda \, dz$$

To perform integrations:
always rewrite charge and current elements *(dQ, dI)* in the form of coordinate elements *(dx,dy,dz* or other*)*, using charge and current densities (λ, σ, ρ and $\boldsymbol{j}$).

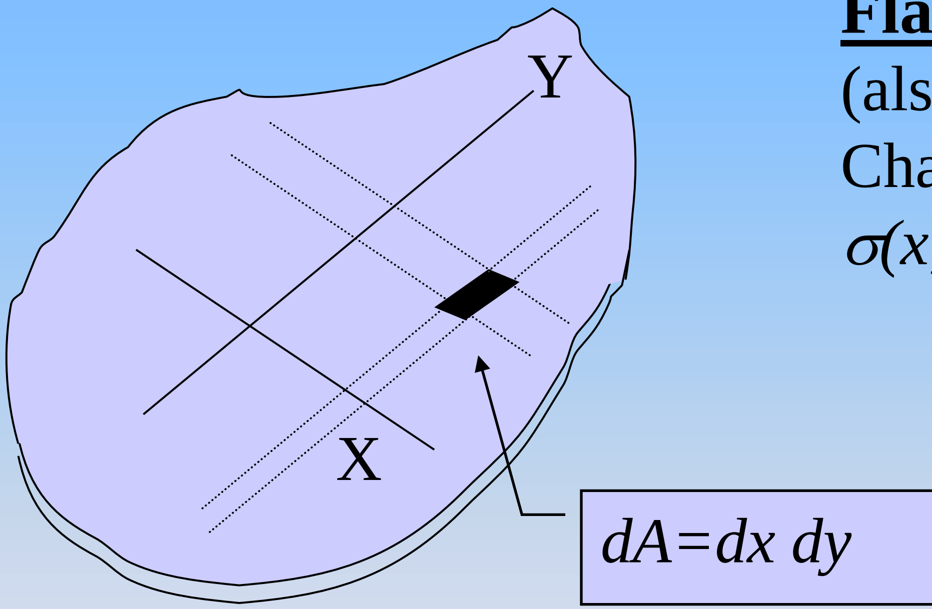
Charge elements: Thin ring



<u>Thin ring</u> (thickness<<*R*)
Charge distribution:
$\lambda(\varphi)$ [C/m]
If homogeneous: λ = const.

$$dQ=\lambda\, R\, d\varphi$$

Charge elements: Flat surface



Flat surface (in general)

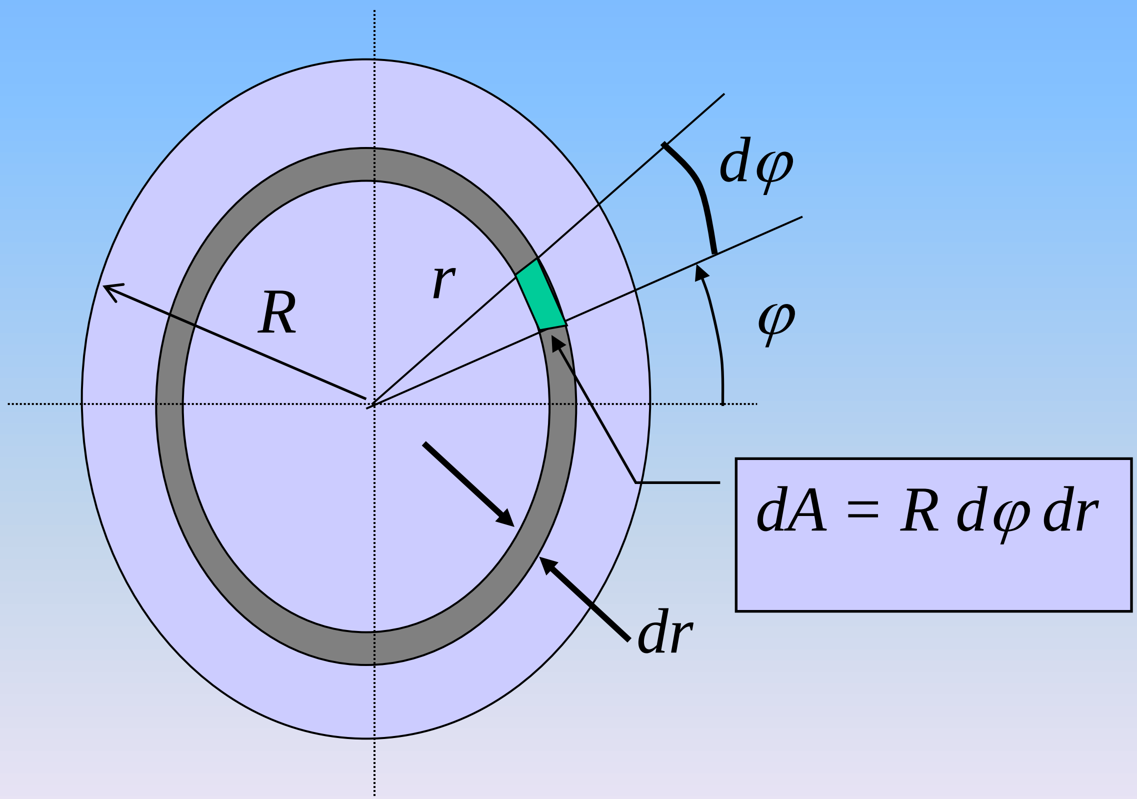
(also for rectangles)

Charge distribution:

$\sigma(x,y)$ [C/m^2]

$$dQ=\sigma(x,y)\,dx\,dy$$

Charge elements: Thin circular disk



Thin circular disk

Charge distribution:

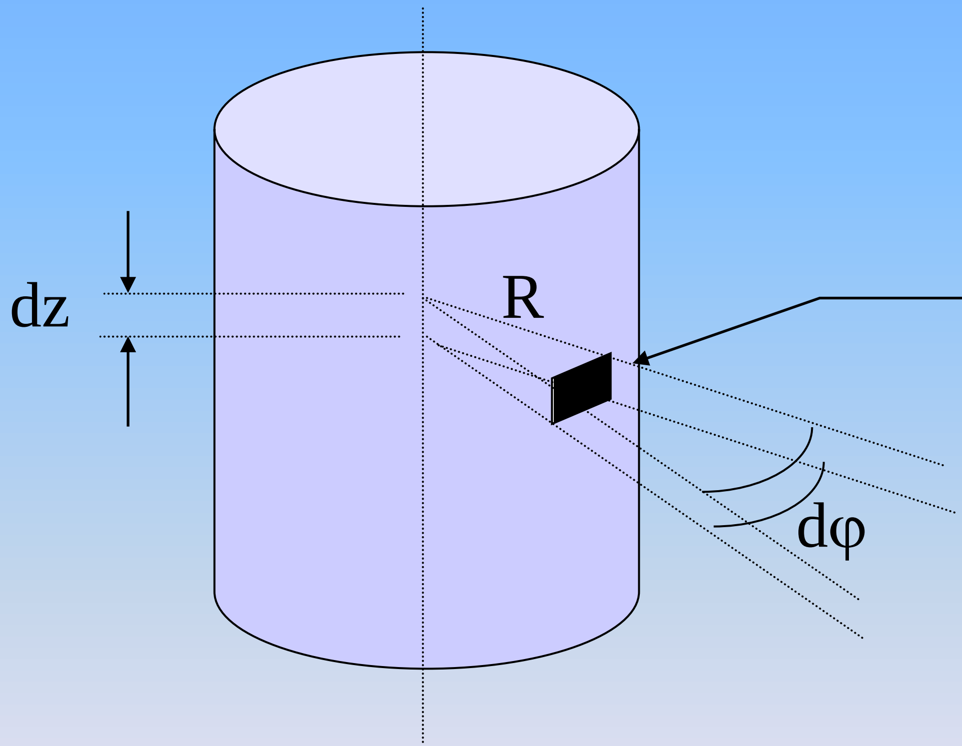
$\sigma(r,\varphi)$ [C/m^2]

$dQ = \sigma(r,\varphi).r\, d\varphi.dr$

If σ = const:

$dQ = \sigma.2\pi\, r.dr$

Charge elements: Cylindrical surface



Cylinder surface

Charge distribution:

$\sigma(\varphi,z)$ [C/m^2];

Radius R

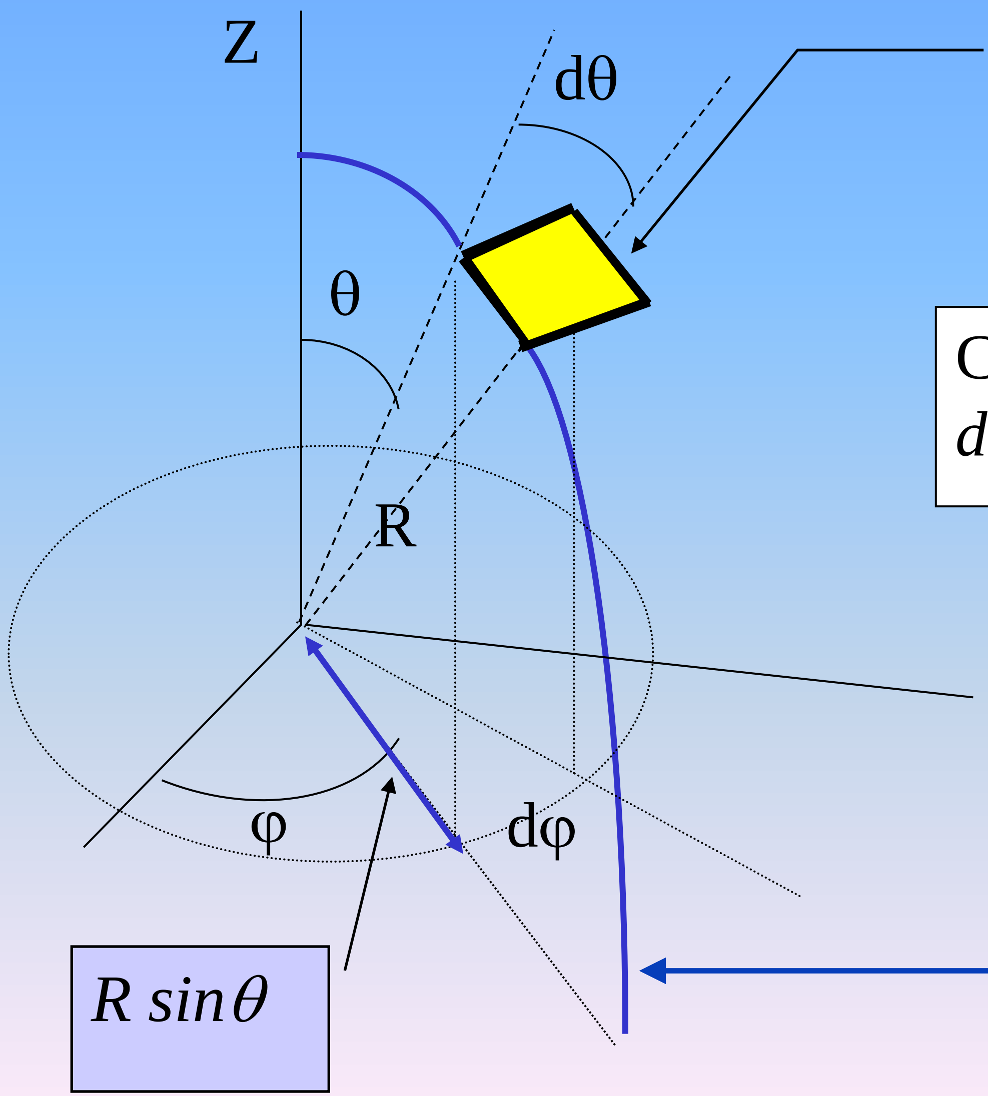
$dA = R\, d\varphi\, dz$

$dQ = \sigma(\varphi,z)\, R\, d\varphi\, dz$

If σ=const:

$dQ = \sigma\, 2\pi R\, dz$

Charge elements: Spherical surface



Surface element

$dA=(Rd\theta)(R\sin\theta\, d\varphi)$

Charge element

$dQ=\sigma\,(R\, d\theta)(R\,\sin\theta\, d\varphi)$

Part of:

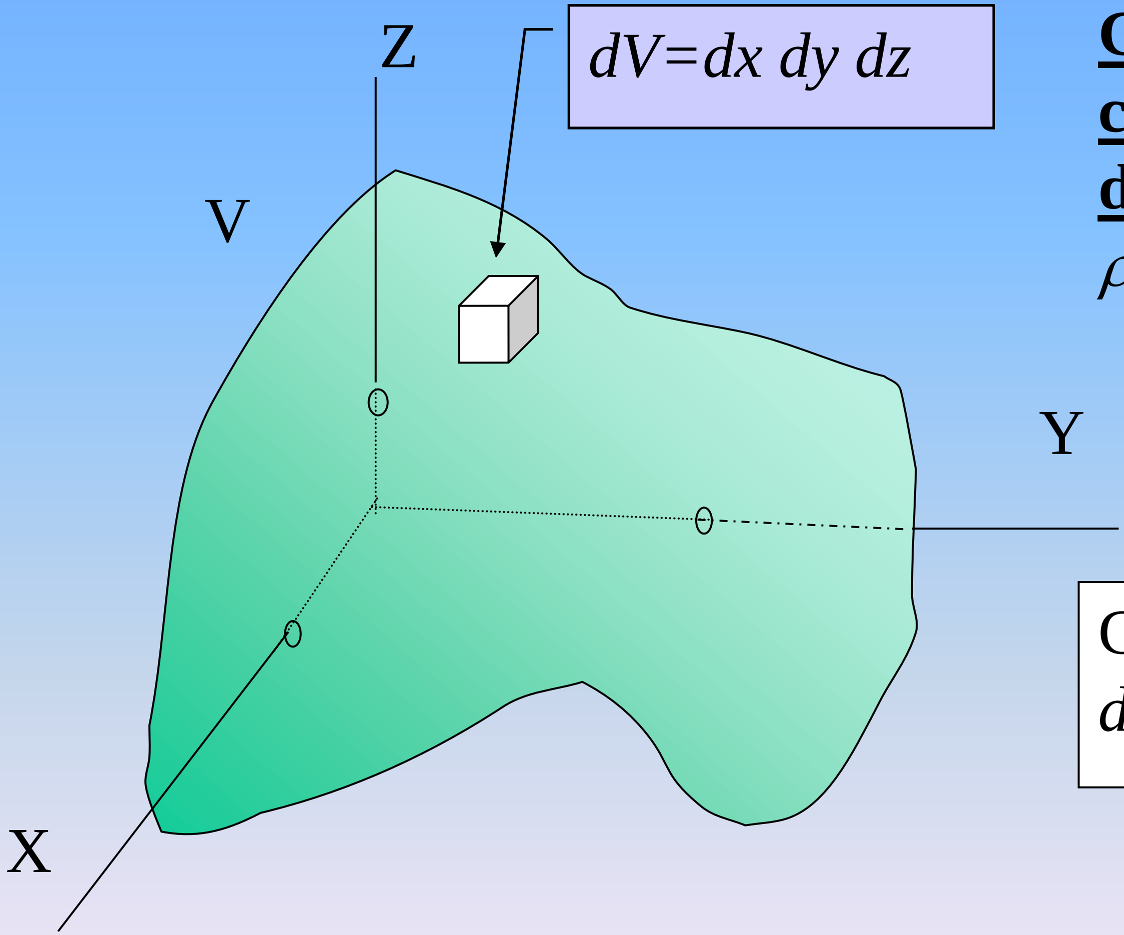
Spherical surface:

Radius: R

Charge distribution:

$\sigma(\theta,\varphi)$ [C/m^2]

Charge elements: Spatial distribution



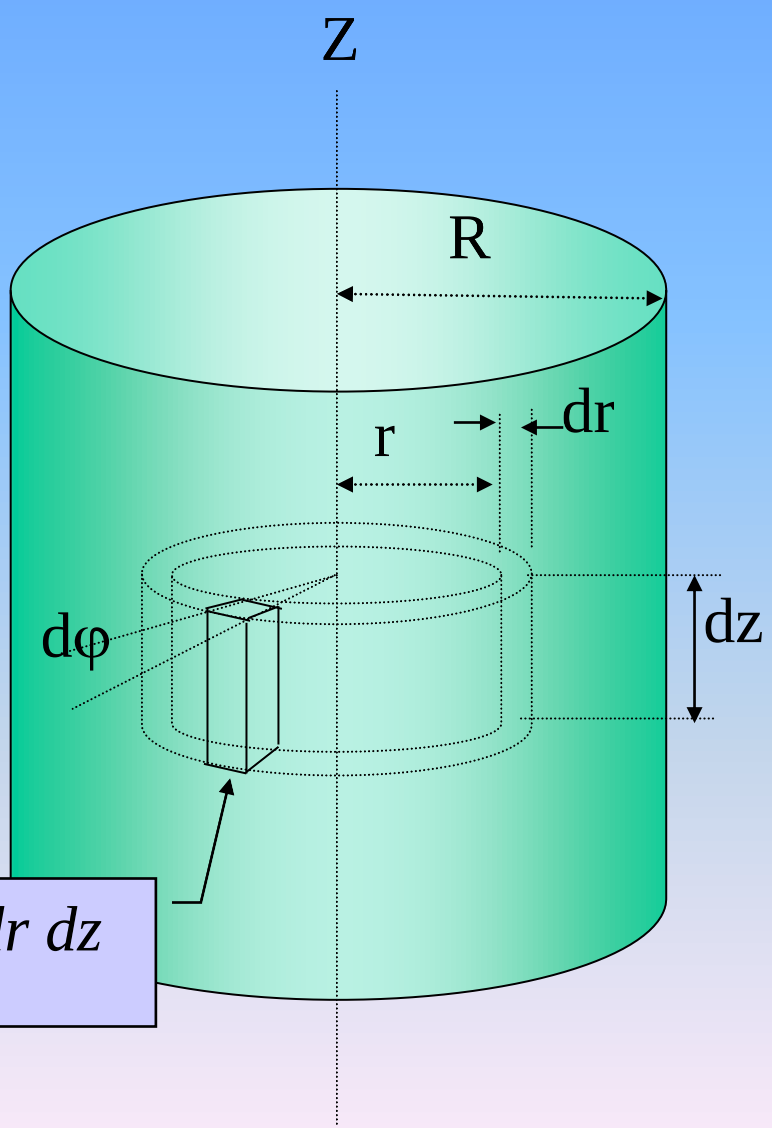
<u>General spatial charge distribution</u>:

$\rho(x,y,z)$ [C/m^3]

Charge element

$dQ=\rho(x,y,z).dxdydz$

Charge elements: Cylindrical volume



$dV = r d\varphi \, dr \, dz$

Cylindrical spatial charge distribution:

$\rho(r,\varphi,z)$ [C/m^3]

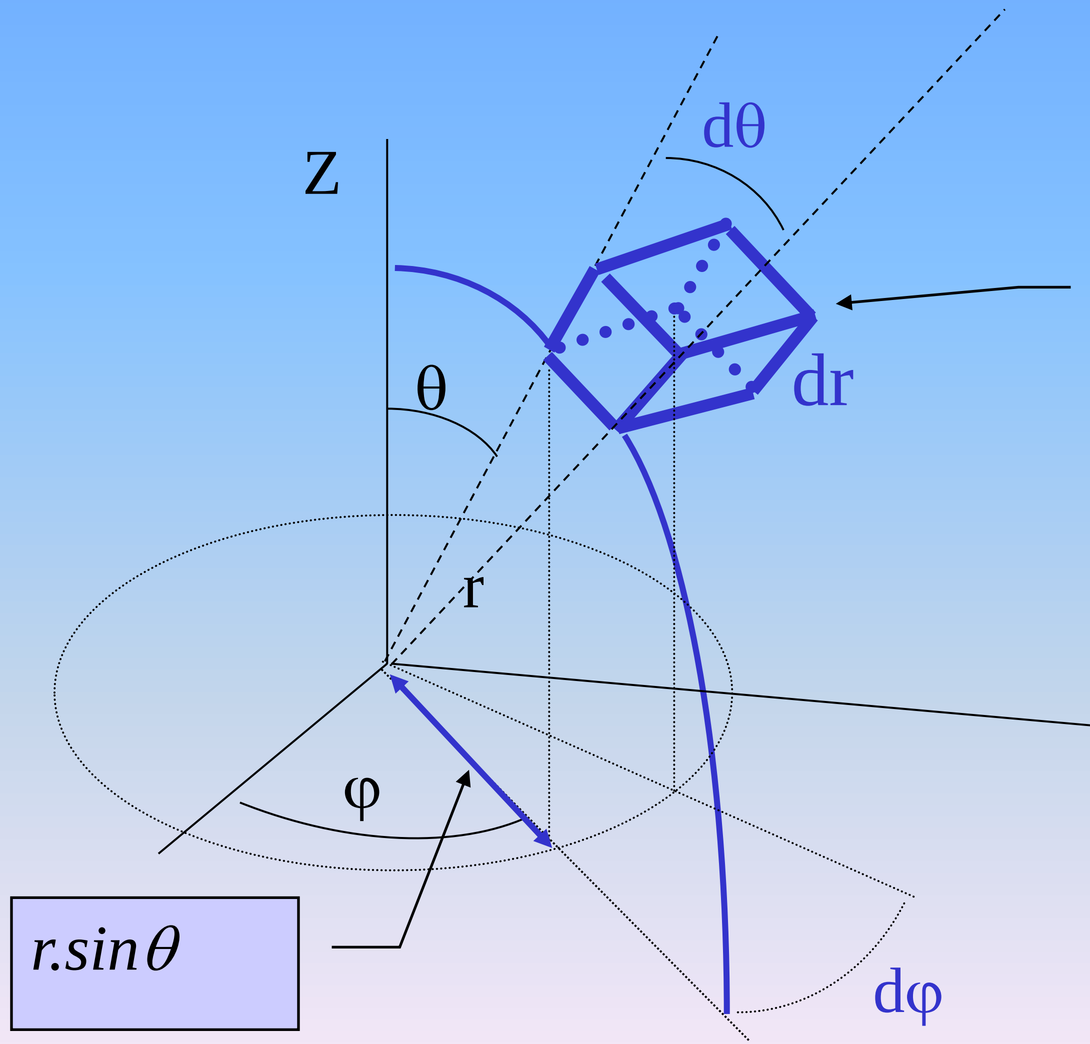
Charge element

$dQ = \rho \, r d\varphi \, dr \, dz$

If ρ independent of φ:

$dQ = \rho \, 2\pi r \, dr \, dz$

Charge elements: spherical distribution



Spherical charge distribution

$\rho(r,\theta,\varphi)$ [C/m^3]

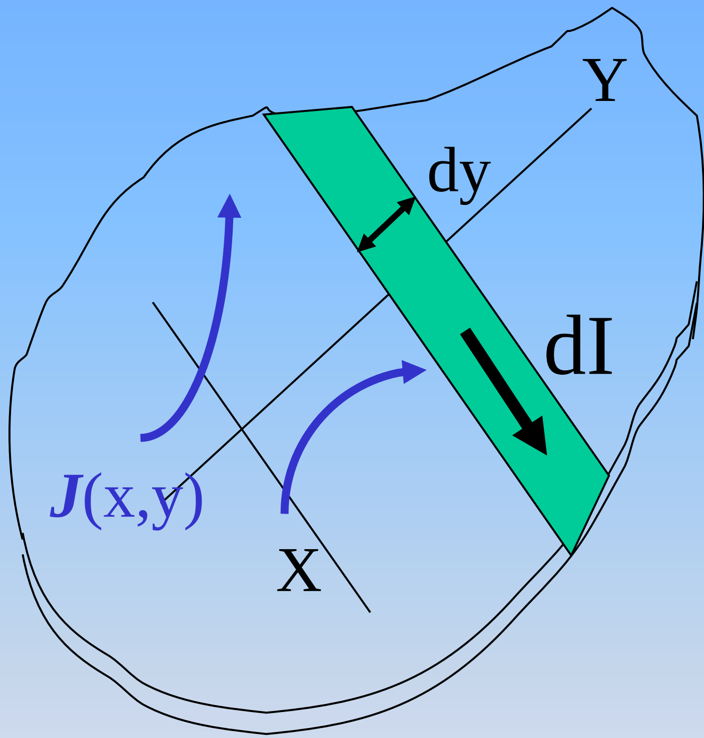
Volume element
$dV=(rd\theta)(r\sin\theta\,d\varphi)\,dr$

Charge element
$dQ=\rho\,(rd\theta)(r\sin\theta\,d\varphi)\,dr$

If ρ independent of θ and φ :

$dQ=\rho\,4\pi r^2\,dr$

Current elements: Flat surface



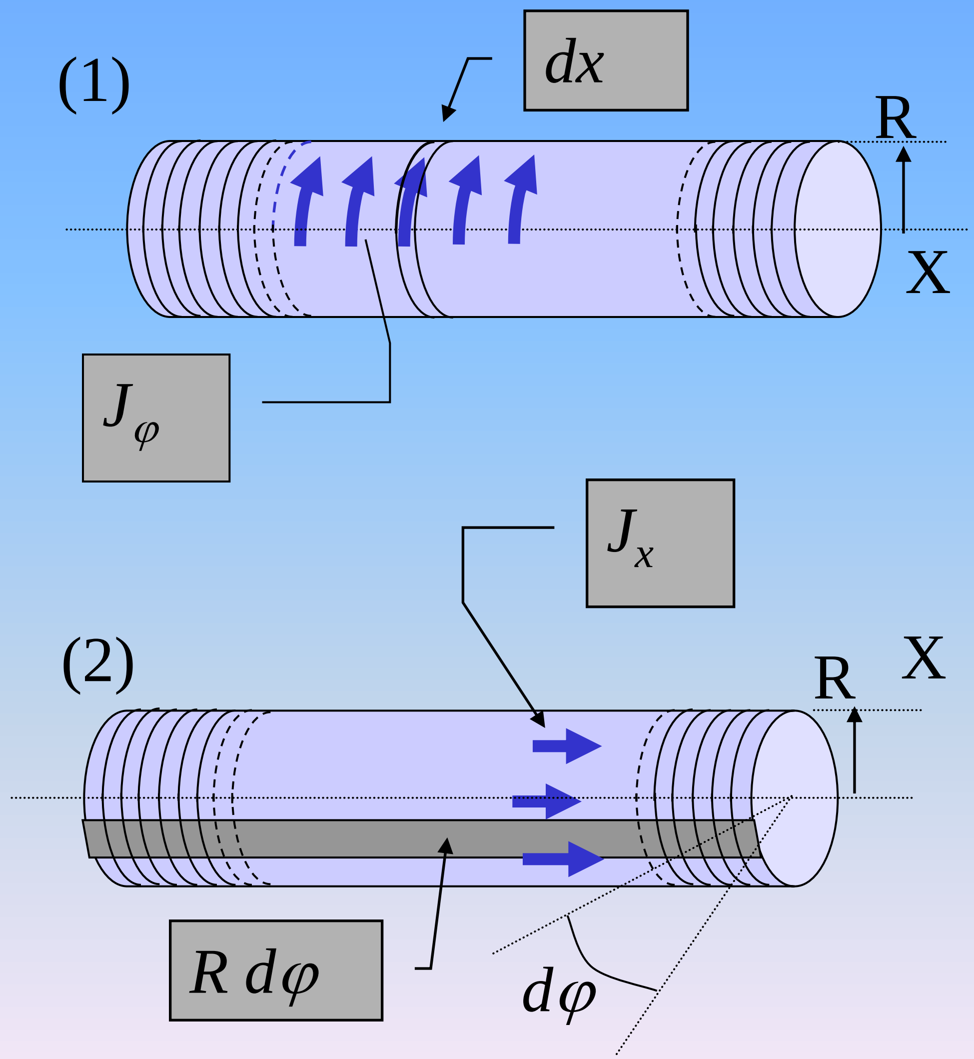
General flat surface with current density:
J **(x,y) , in [A/m]**

Contribution to current element *dI* through strip *dy* in +X-direction:

Current element
$$dI = (\mathbf{J} \bullet \mathbf{e}_x)\, dy = J_x . dy$$

J_x current density [A/m]

Current elements: solenoid surface



Solenoid surface current

N windings, length *L*;

Current densities:

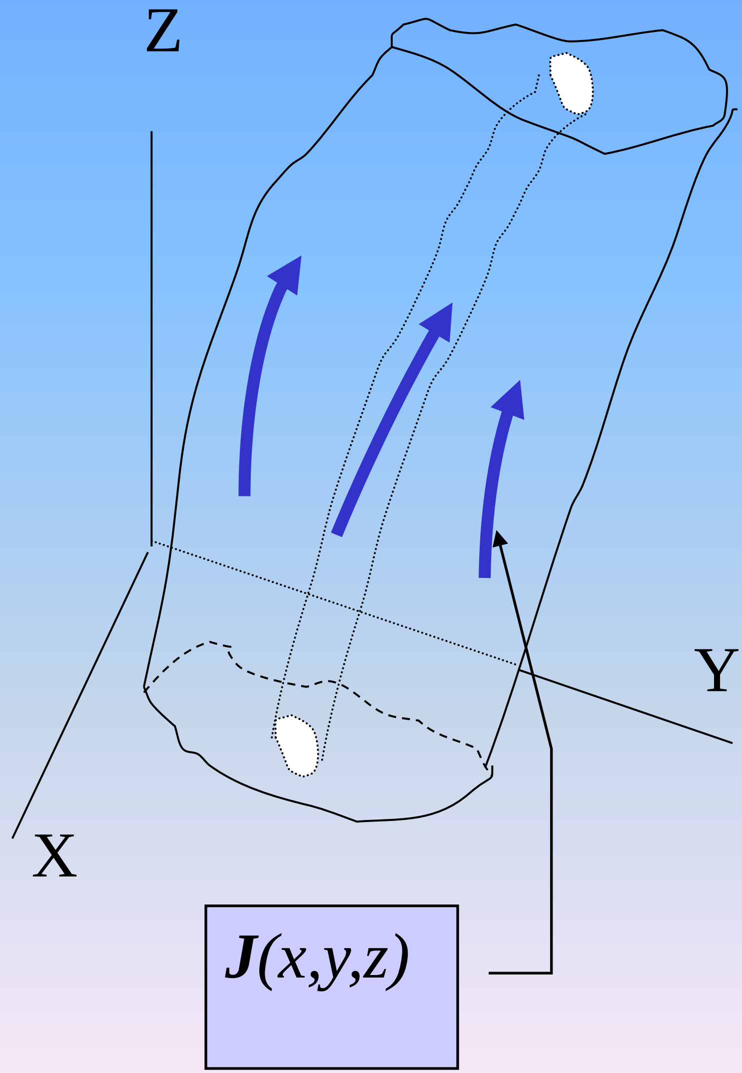
(1): j_φ tangential [in A per m length]

$$dI = J_\varphi\, dx = NI\, dx/L$$

(2): j_x parallel to X-as [in A per m circumference length]

$$dI = J_x\, R.d\varphi$$

Current elements: General current tube



<u>General current tube:</u>

J*(x,y,z)* : [A/m^2] = volume current through material,

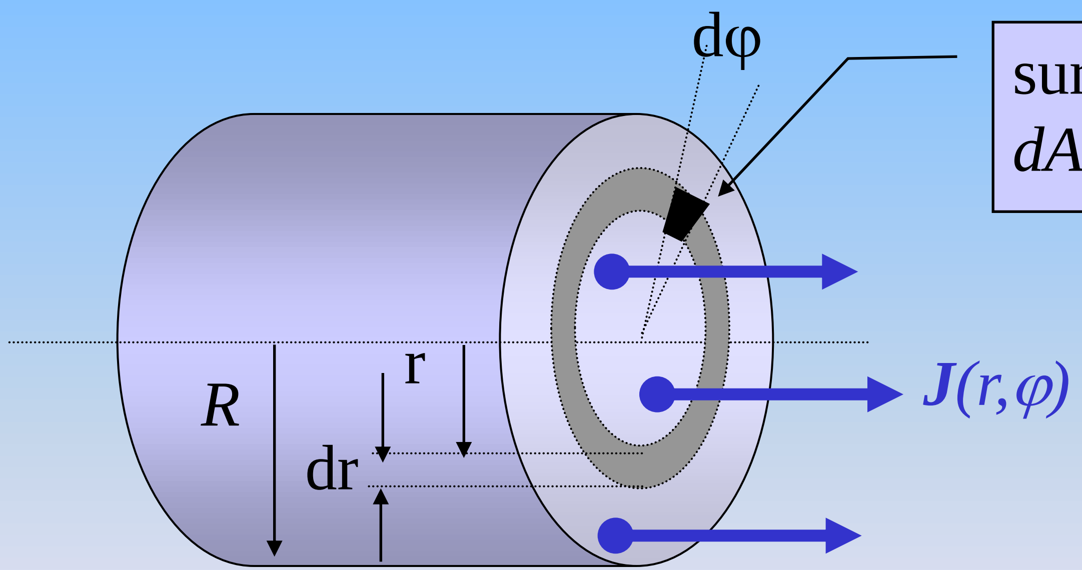
<u>current element</u> dI_z = contribution to current in Z-direction :

$$dI_z = \mathbf{J}(x,y,z) \bullet \mathbf{e_z}\, dxdy$$
$$= J_z\, dxdy$$

Current elements: Thick wire

Cylinder

Current density [A/m^2] through material, parallel to symmetry axis



surface element: ring element

$dA = r d\varphi \, dr$

Current density

$dI = J(r,\varphi).r d\varphi.\, dr$

the end