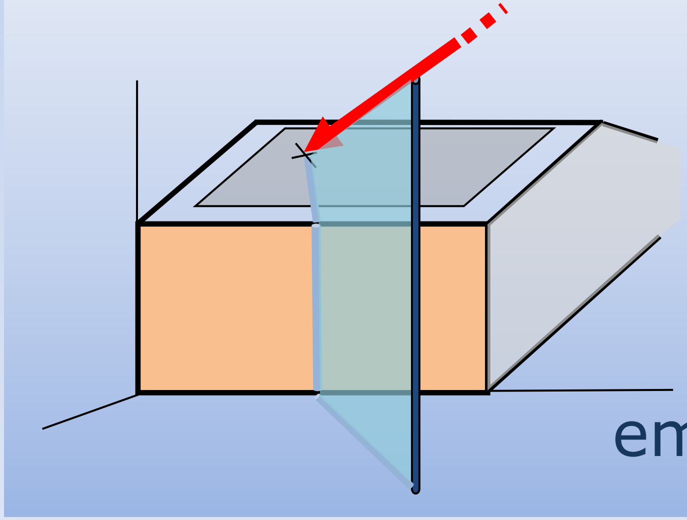
Solar Panel Irradiation

Energy efficiency of solar panels with shadows and atmospheric effects



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2017

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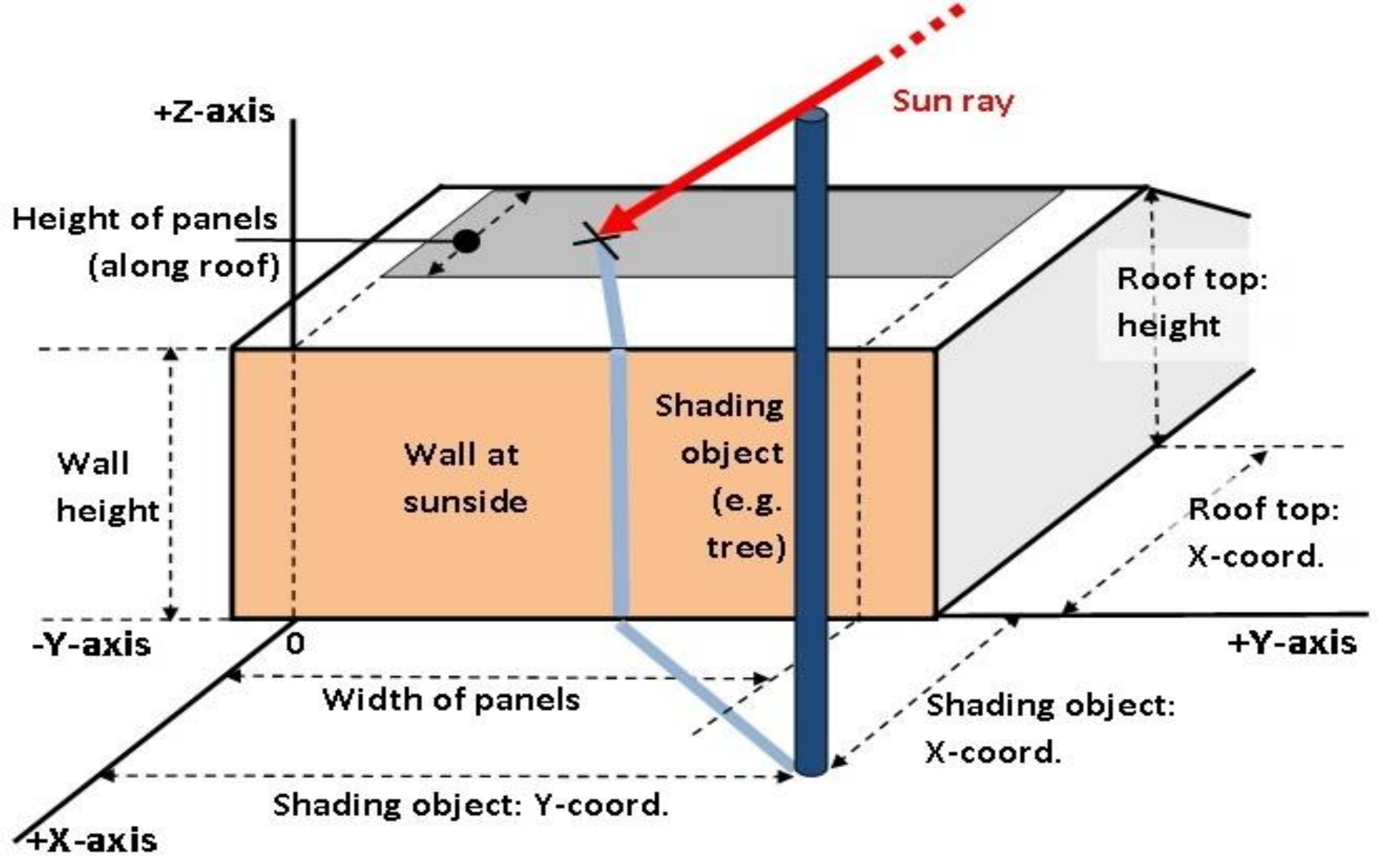
Solar Panel Irradiation and Efficiency



Solar Panel Irradiation and Efficiency

Contents:

1. Computer simulation:
   - Shadow tracks of nearby objects (e.g. trees) on an oblique roof.
   - Irradiation efficiency and energy.

Online program: ▶

2. The physical mathematics behind the calculations:
   - Irradiation angles
   - Irradiation efficiency
   - Effects of cloudy skies and diffuse scattering
   - Energy output

Solar Panel Irradiation and Efficiency

1. **Local Times**, corrected for summertime (?) and time zone (w.r.t. GMT)

2. **Irradiation angles** and **Solar Ray Tracking:**
   zenith (**elevation**, **altitude**) and **azimuth**,
   corrected for wobbling and axis tilting of the earth,
   as functions of Julian date and time (“Hour Angle”).

3. **Exact sunrise and sunset times**.

4. **Irradiation efficiency** compared with normal sun ray incidence.

5. **Shadow lines** caused by shading objects on the panels.

6. **Atmospheric effects** (transmission, refraction, clearness, clouds…).

7. **Energy output** (kWh/year).

- Corrections for:
  - **Equation of time** (eccentricity of orbit and tilt of axis)
  - **Atmospheric refraction and extinction** (scattering and absorption)
  - **Sunrise and sunset shifts** due to finite diameter of solar disk

Solar Panel Irradiation and Efficiency

Literature:

- For solar angles and times: best entrance: see Wikipedia
- Guttman,A., “Extinction coefficient measurements on Clear Atmospheres and Thin Cirrus Clouds", Appl.Opt. 7, 12, 1968, 2377-2381
- M.L. Roderick, "Methods for Calculating Solar Position and Day Length ..", RMTR 137, ISSN0729-3135, 1992, Western Australian Dept. of Agriculture, Perth, Australia.
- B.H. Liu, R.C. Jordan, “The Interrelationship and Characteristic Distribution of Direct, Diffuse and Total Solar Radiation”, Solar Energy, 4, 1960, 1-9.
- For shadow projection and sun ray tracking:

  any Analytical Geometry textbook.

1. Local Times and “Hour Angle”

Definitions:

- ***LT*** : Local Time : (h)

  actual time at your position (as read from your watch)

- ***LST*** : Local Solar Time : (h)

  actual time if the sun’s highest altitude (due South), locally at your position, were at 12:00 h

- ***DT*** : Time difference with GMT : (h)

  (GMT = Greenwich Mean Time)

- ***LSTM*** : Local Standard Time Meridian: (^{0})

  $LSTM = DT * 360^0 / 24\ h$

1. Local Times and "Hour Angle"

Local Solar Time (LST) : (h)

(if your sun were due South at 12:00 h : highest elevation or altitude)

Correction of Local Time (LT: as read from your watch) for:

- Equation of Time ***EoT*** (correct for tilting and wobbling of the earth)
- Position ***LSTM - ψ*** in the time zone

LSTM = local standard time meridian (diff. with GMT, in ^{0})

ψ = longitude (in ^{0} ; +/- on eastern/western hemisphere)

$$LST = LT + \frac{EoT}{60} - \frac{24}{360}(LSTM - \psi)$$

Hour angle (HRA): (^{0})

Local Solar Time (*LST*) expressed in degrees

Hour angle = 0 at solar 12:00 h,

<0 , >0 in morning, afternoon

$$HRA = \frac{360}{24}(LST - 12)$$

1. Local Times and "Hour Angle"

Equation of time (EoT): (in min.)

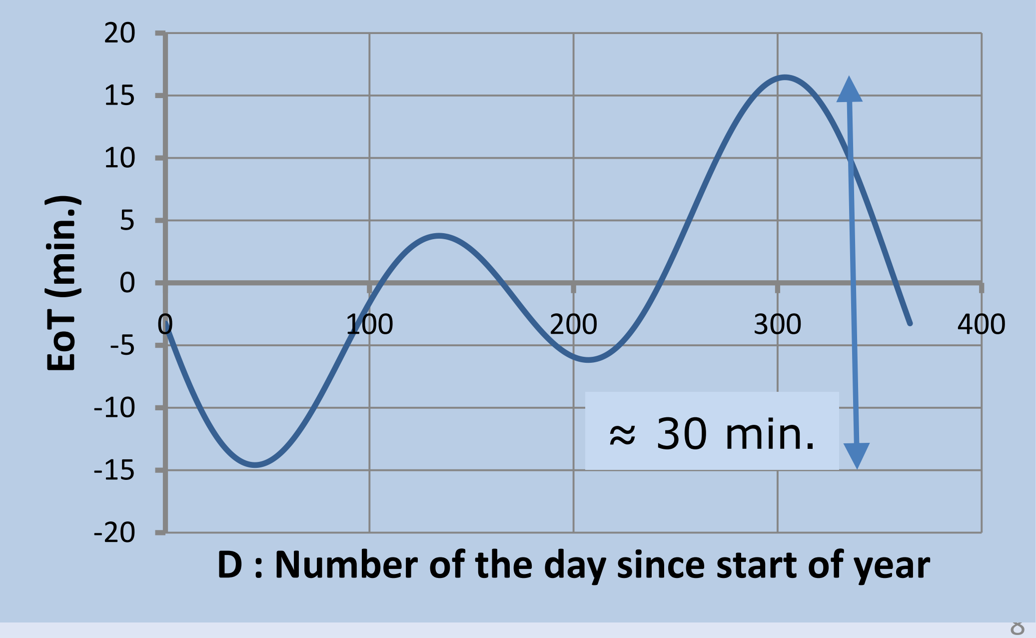
Empirical relation; corrects for :

- wobbling of the earth's orbit ,
- the tilt of the earth's axis.

$$EoT = 9.87\sin(2\beta) - 7.53\cos\beta - 1.5\sin\beta$$

$$\beta = \frac{360}{365}(D - 81)$$

D = number of the day since the start of the year



2. Irradiation angles: Solar Ray Tracking

Julian Day Number (JDN)

$$JDN = d + \frac{153(M+2)}{5} + 365.Y + \frac{Y}{4} - \frac{Y}{100} + \frac{Y}{400} - 32045;$$

$$M = m + 12.A - 3 \quad ; \quad Y = y + 4800 - A \quad ; \quad A = \frac{14-m}{12}$$

y = year (A.D.) ; *m* = month ; *d* = day

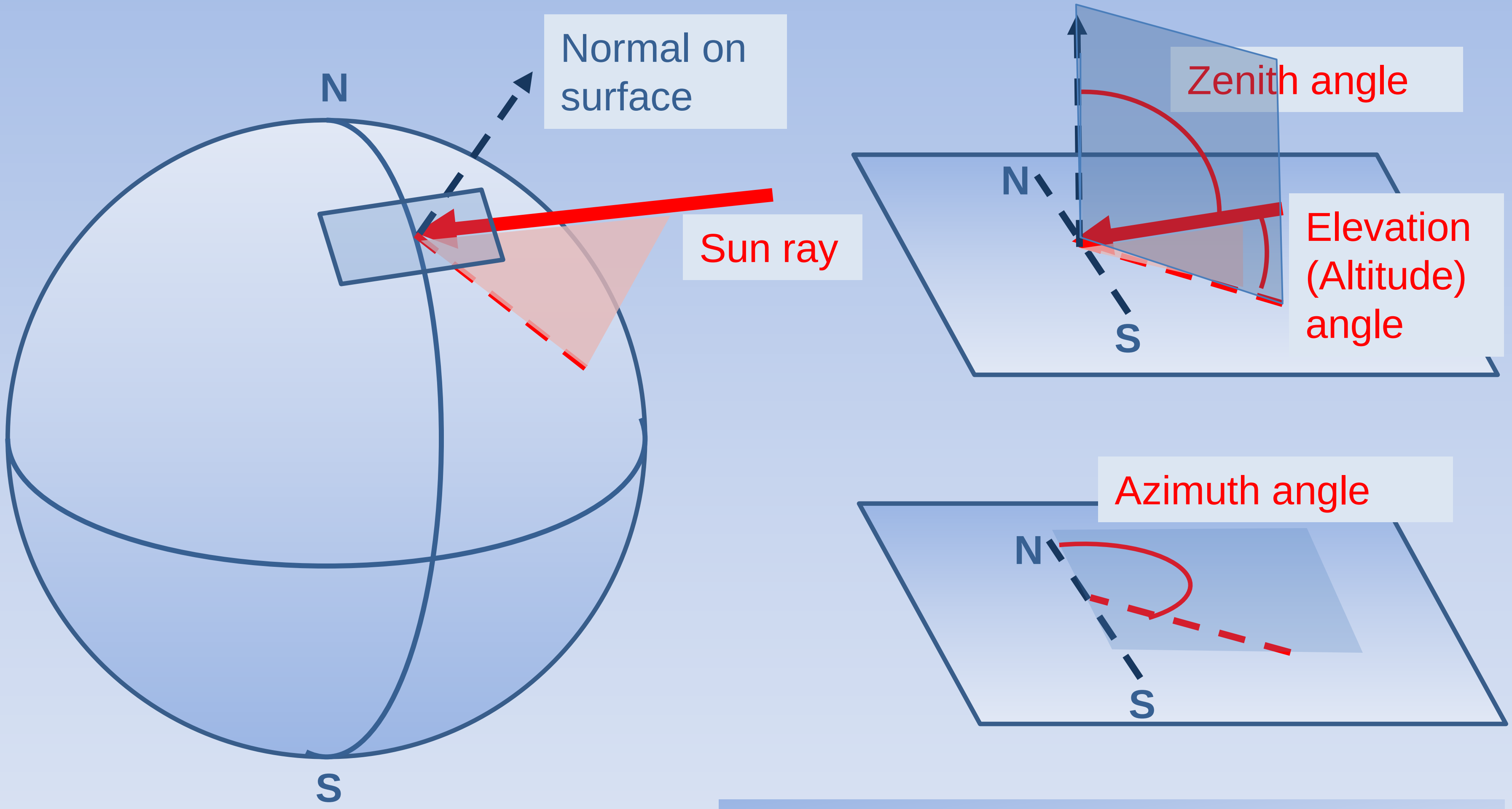
→ All these divisions have to be truncated:
fractional parts dropped (to account for leap-days)

Julian Day Time (JD)

$$JD = JDN + \frac{hour - 12}{24} + \frac{min}{24.60} + \frac{sec}{24.3600}$$

1 Jan. 2000 AD., 12:00 h => JD = 2451545.0
24 Dec. 4712 BC., 12:00 h => JD = 0

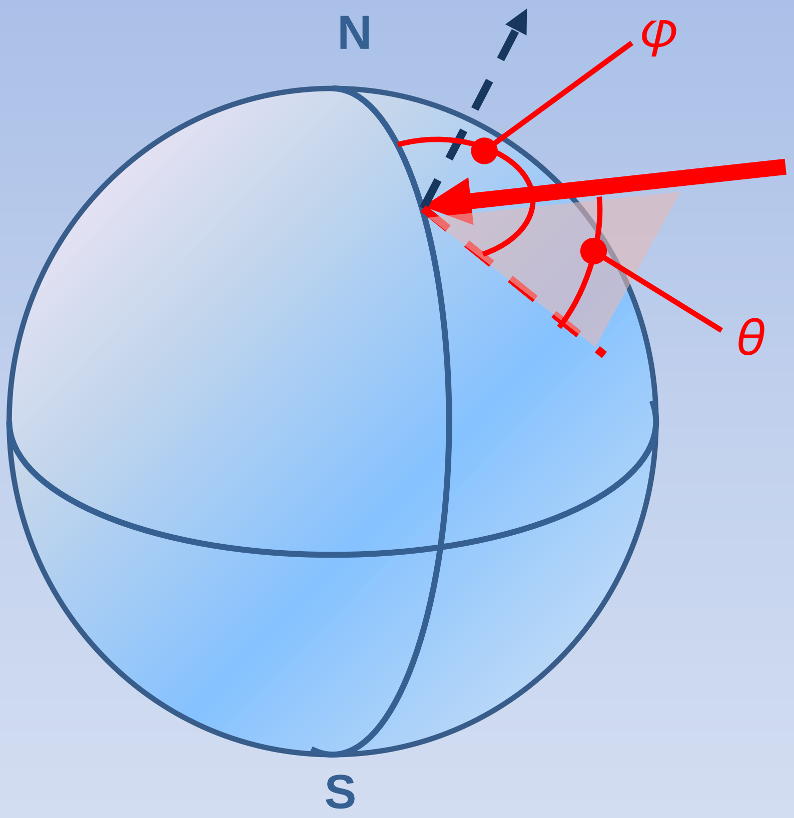
2. Irradiation angles: Solar Ray Tracking



Local normal vector points to local **zenith**

Elevation (altitude) angle: θ
Azimuth angle: φ

2. Irradiation angles: Solar Ray Tracking



θ : Elevation (altitude) angle
φ : Azimuth angle

$$\sin\theta = \cos(HRA).\cos\delta.\cos\xi + \sin\delta.\sin\xi$$

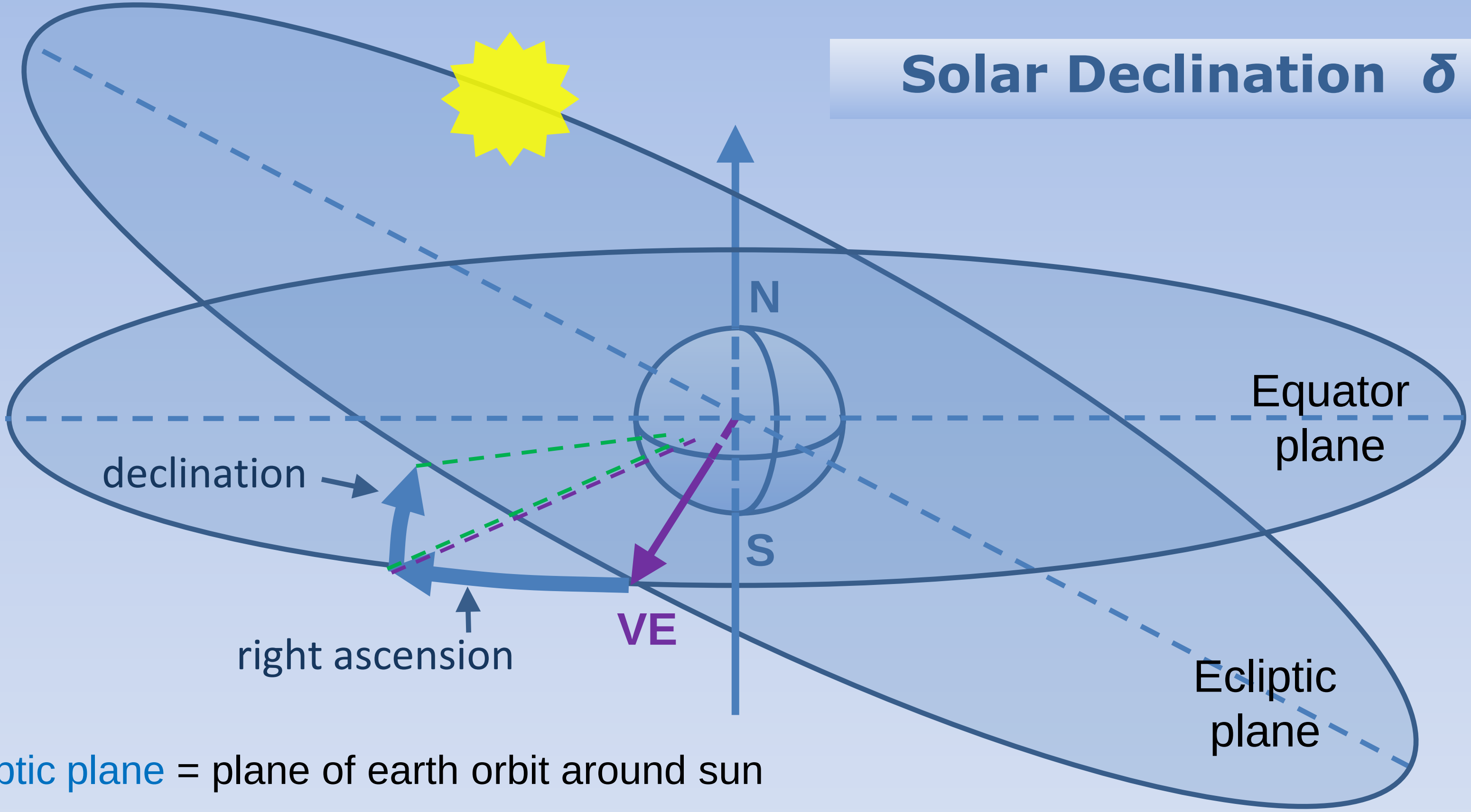
$$\cos\varphi = \frac{\sin\delta - \sin\theta.\sin\xi}{\cos\theta.\cos\xi}$$

HRA = Hour Angle (-/+ in morning / evening)
δ = sun declination (--> next screen)
ξ = local latitude (+/-: on N/S hemisphere)
ψ = local longitude (+/-: on E/W hemisphere)

φ is measured from North direction
(N, E, S, W : => *φ* = *0,* 90, 180, 270^0)
φ from formula is to be interpreted as the angle < or > 180^0 when *HRA* < or > 0
ζ = zenith angle (= 90° - *θ*)

2. Irradiation angles: Solar Ray Tracking

Solar Declination δ



Ecliptic plane = plane of earth orbit around sun

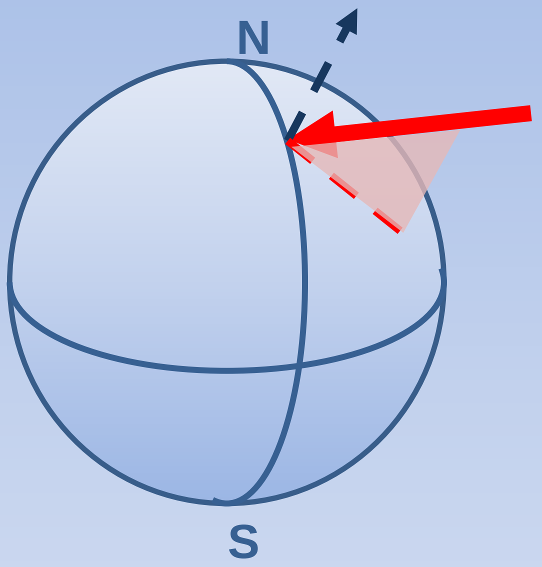
VE = vernal equinox (21-22 March)

Declination and Right ascension : angles as seen from earth

Right ascension (in h) = Longitude (in ^{0})

Solar declination varies with the seasons (between -23.44^o and +23.44^0)

2. Irradiation angles: Solar Ray Tracking



Solar Declination δ given by:

$$\sin \delta = \sin \varepsilon . \sin \lambda$$

ε = **obliquity of the ecliptic** :
approximation:

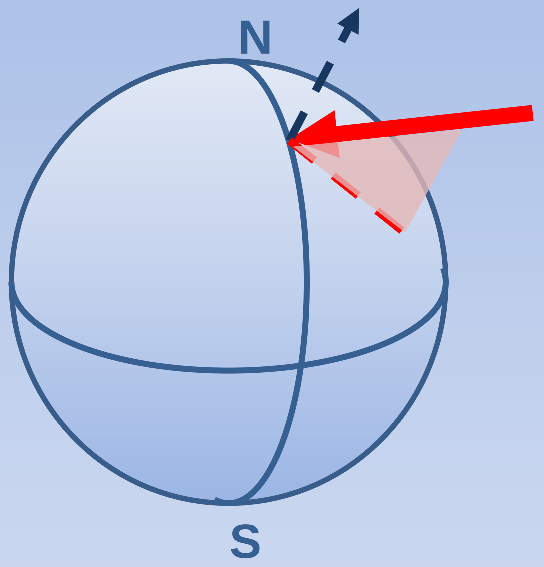
$\varepsilon = 23.439^0 - (4.10^{-7})^0 . n$

n = Julian day number after 1 Jan. 2000
= $JD - 2451545.0$

λ = **solar ecliptic longitude:**

$\lambda = L + 1.915^0 . \sin g + 0.020^0 . \sin 2g$
$L = 280.460^0 + 0.9856474^0 . n$
$g = 357.528^0 + 0.9856003^0 . n$

3. Sunrise and Sunset



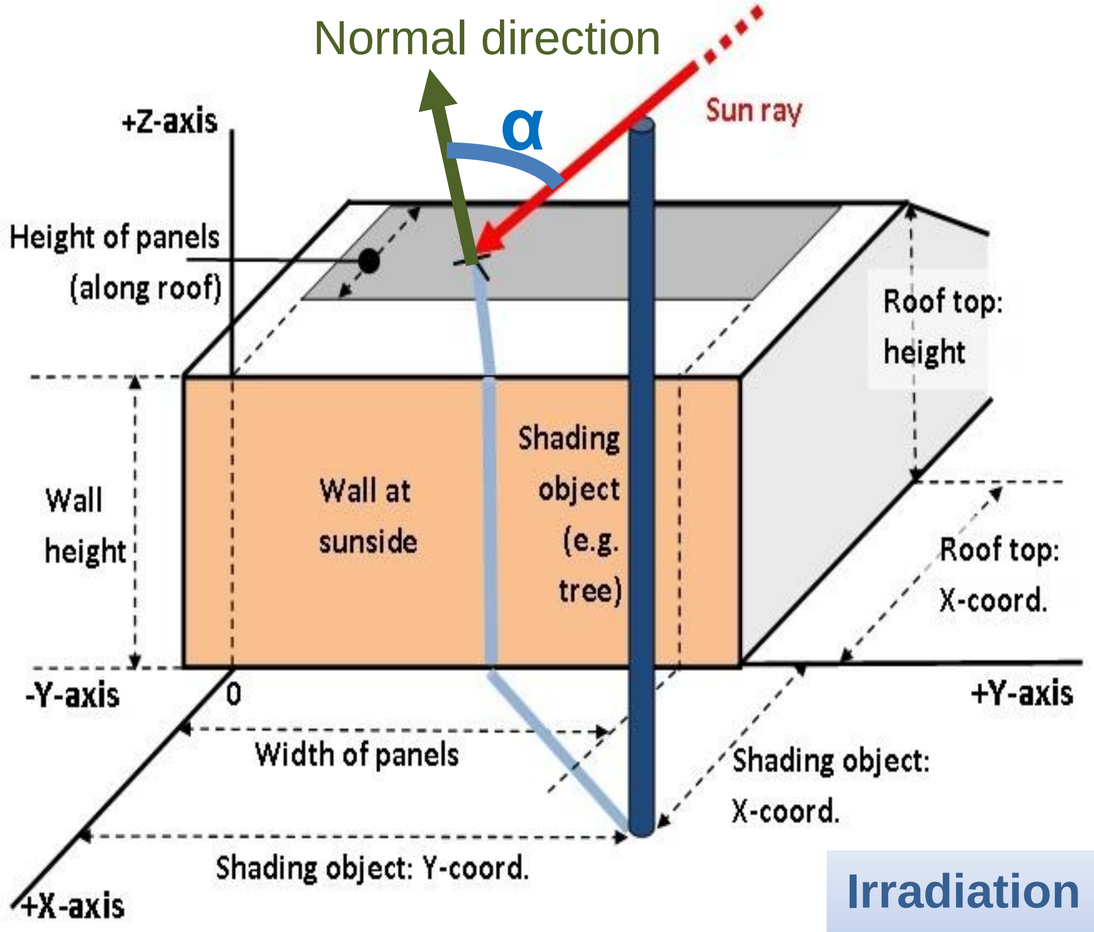
Sunrise and sunset times:

Calculated using linear interpolation of
solar zenith angles
at subsequent times of the day,

to find times at which solar zenith angle = 90.833^0
(or elevation = -0.833^0).

This correction of 0.833^0 is applied
to account for refraction and
the finite dimensions of the apparent solar disk.

4. Irradiation efficiency



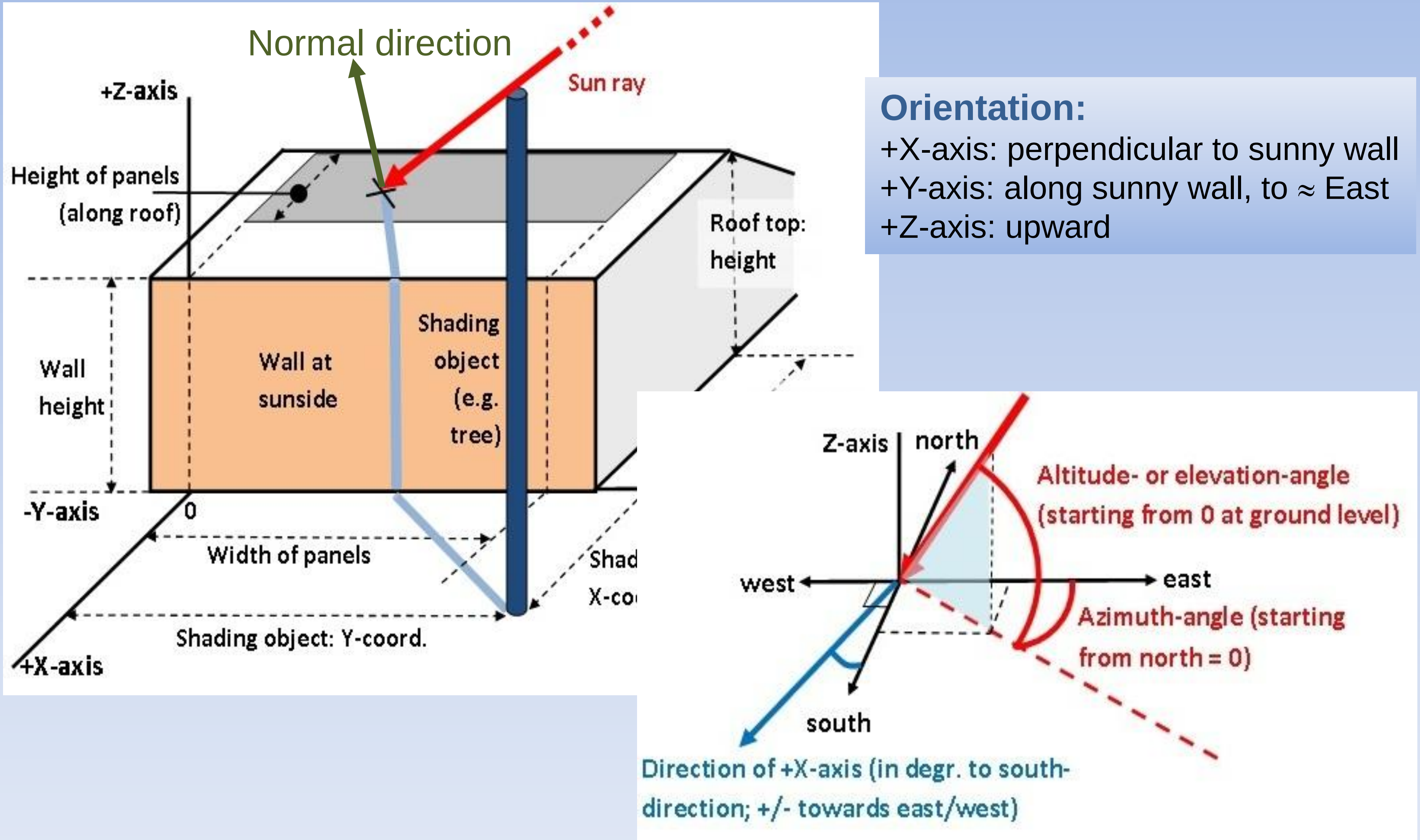
Irradiation efficiency:

defined as:

Cosine of the angle α between **solar ray** and **normal direction** (perpendicular) on roof plane.

Irradiation efficiency:
= 1 at perpendicular incidence of sun ray
= 0 at parallel incidence

4. Irradiation efficiency



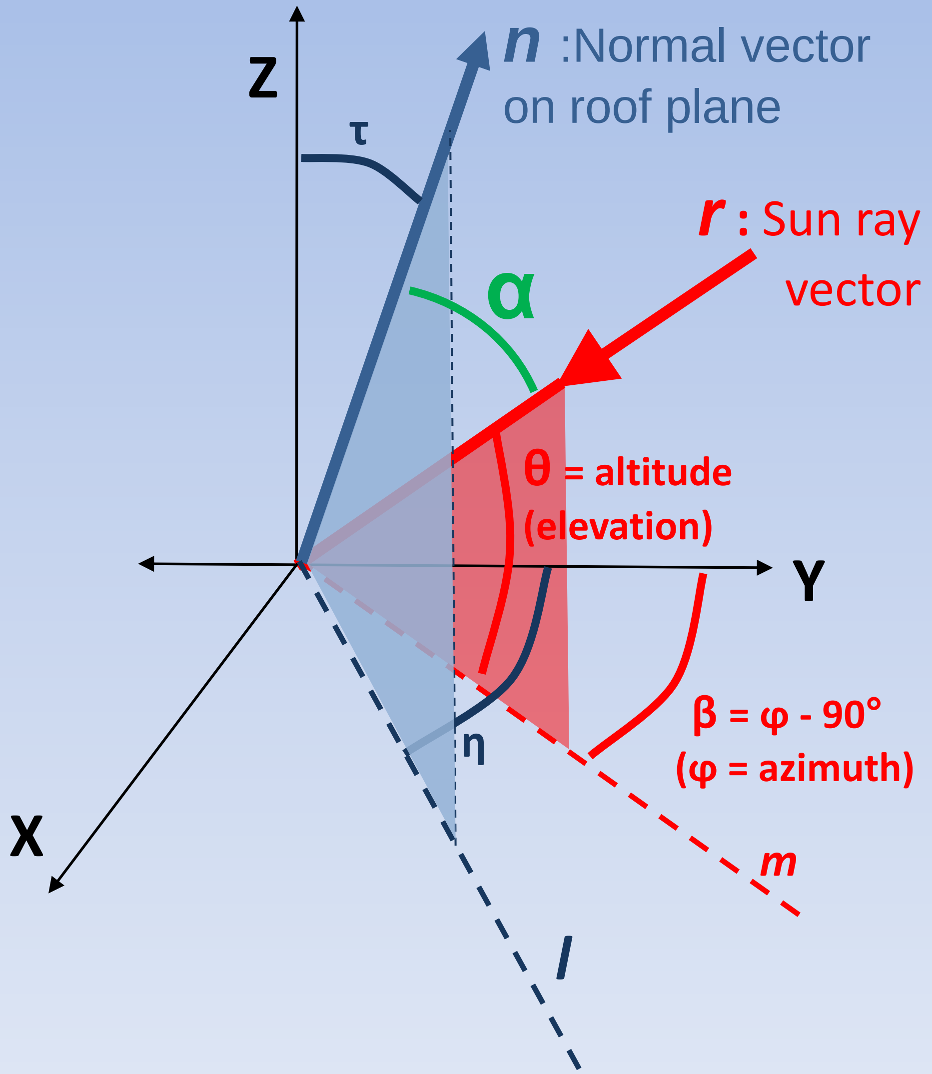
Orientation:

+X-axis: perpendicular to sunny wall

+Y-axis: along sunny wall, to ≈ East

+Z-axis: upward

4. Irradiation efficiency



Sun ray vector = ***r***

Normal vector on roof plane = ***n.***

Lines ***l*** and ***m***, and angles ***η***, ***β*** and ***φ*** : in XY-plane

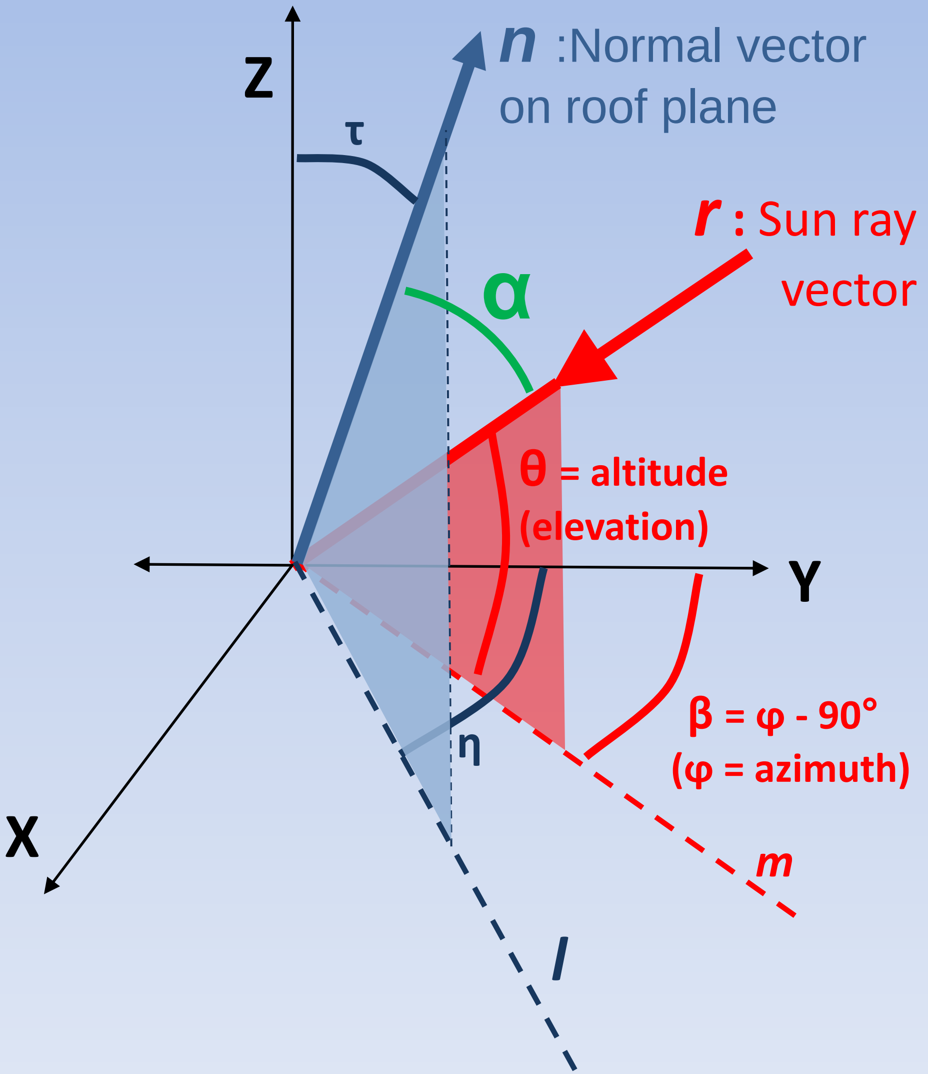
Irradiation efficiency *E* :

$$E = \cos \alpha$$

$E = 1$ if $\alpha = 0^0$

$E = 0$ if $\alpha = 90^0$

4. Irradiation efficiency



Irradiation efficiency *E* :

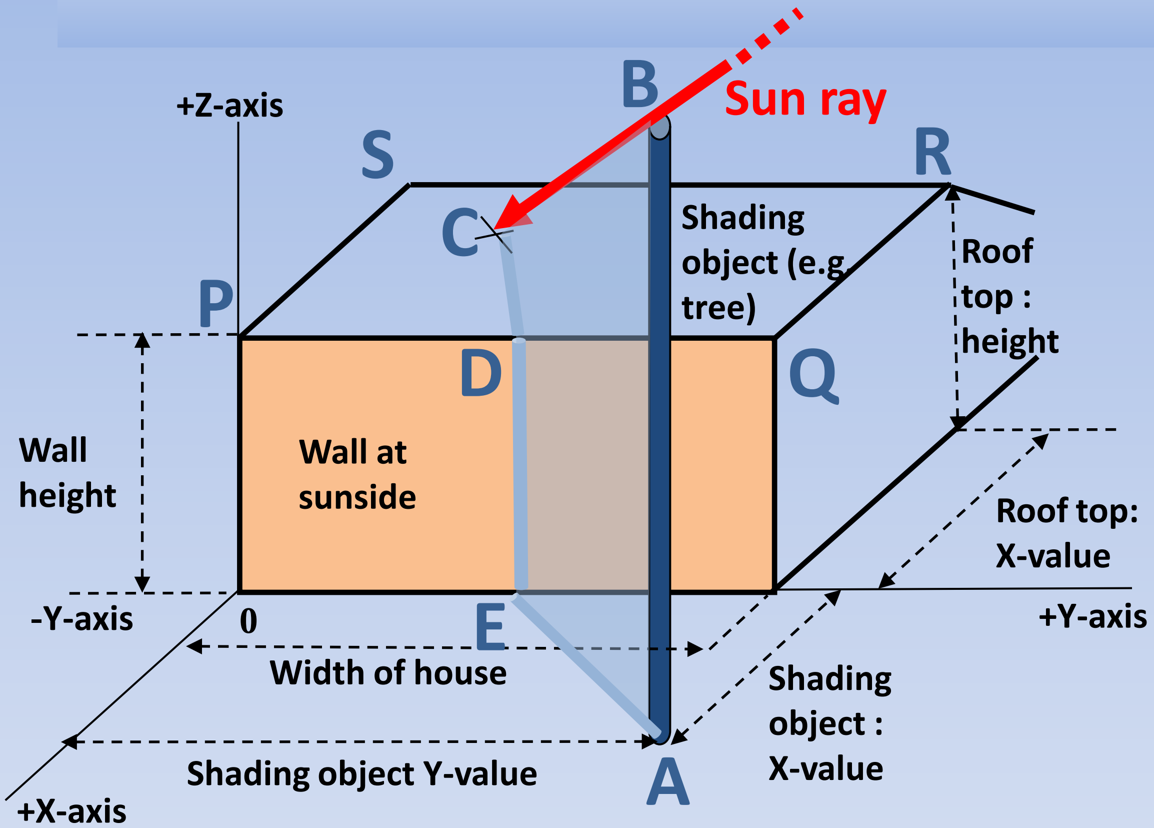
$$E = \cos\alpha = (\boldsymbol{n} \cdot \boldsymbol{r})$$

Assumed: ***n*** and ***r*** have length = 1.

Write vectors ***n*** and ***r*** in X,Y,Z-components and calculate **cos *α*** = (***n* ▪ *r***)

$$E = \cos\theta.\sin\beta.\sin\tau.\sin\eta$$
$$+\cos\theta.\cos\beta.\sin\tau.\cos\eta$$
$$+\sin\theta.\cos\tau$$

5. Shadow lines of shading objects



Calculate:

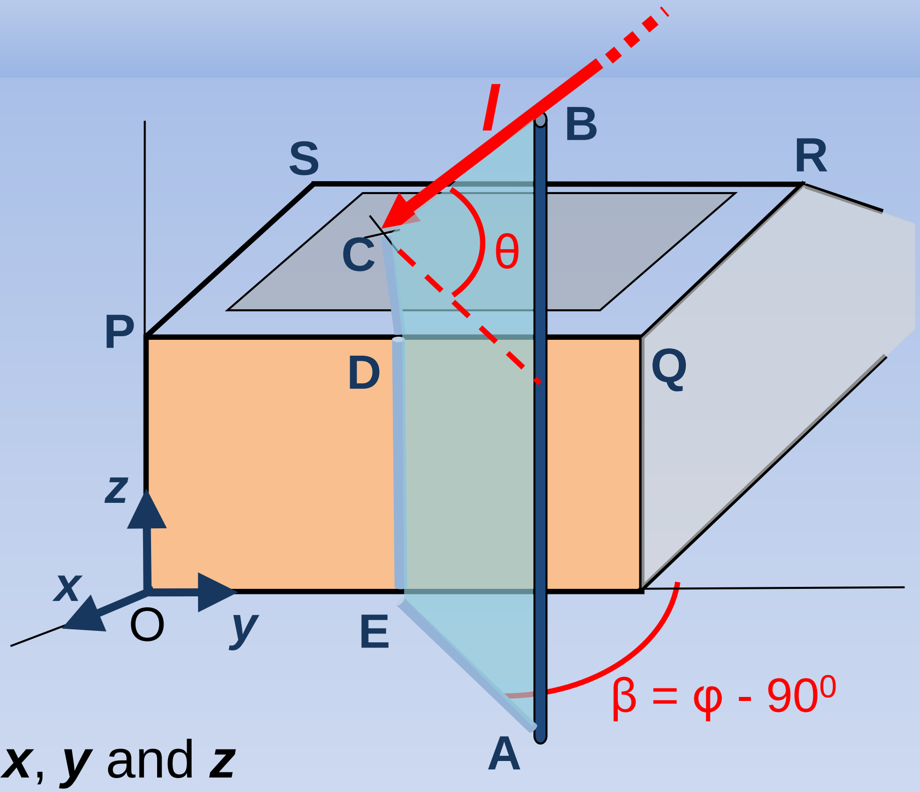
1. Projection of sun ray through top of shading object onto (extended) roof plane.
2. Is crossing point C within roof boundaries?

- Does roof plane PQRS intersect with shadow plane ABCDE?
- (Intersection line = CD).

Orientation:

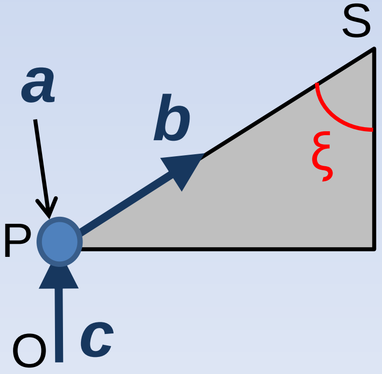
+X-axis: perpendicular to sunny wall
+Y-axis: along sunny wall, to ≈ East
+Z-axis: upward

5. Shadow lines of shading objects



x, ***y*** and ***z*** unit vectors

Roof plane:
cross section:
$\mathbf{a} \perp \mathbf{b}$ and $\mathbf{c}$



Object: top B : $\mathbf{r_B} = (x_0, y_0, z_0)$

Roof: $\mathbf{r_C} = \mathbf{c} + \lambda_1\, \mathbf{a} + \mu_1\, \mathbf{b}$

$\mathbf{a} = \mathbf{y} = (0,1,0)$

$\mathbf{b} = -\sin\xi.\, \mathbf{x} + \cos\xi.\, \mathbf{z}$

$\mathbf{c} = (0,0,z_P)$

Solar ray:

$$\mathbf{l} = \mathbf{r_B} + \tau_1 \left(\cos\theta \sin\beta\, \mathbf{x} + \cos\theta \cos\beta\, \mathbf{y} + \sin\theta\, \mathbf{z}\right)$$

Intersection point C at $\mathbf{r_C}$:
(relative w.r.t. P) for $0 < \xi < 90^0$
is given by:

$\mu_1 = -(x_0 + \tau_1 \cos\theta \sin\beta) / \sin\xi$

$\lambda_1 = y_0 + \tau_1 \cos\theta \cos\beta$, with

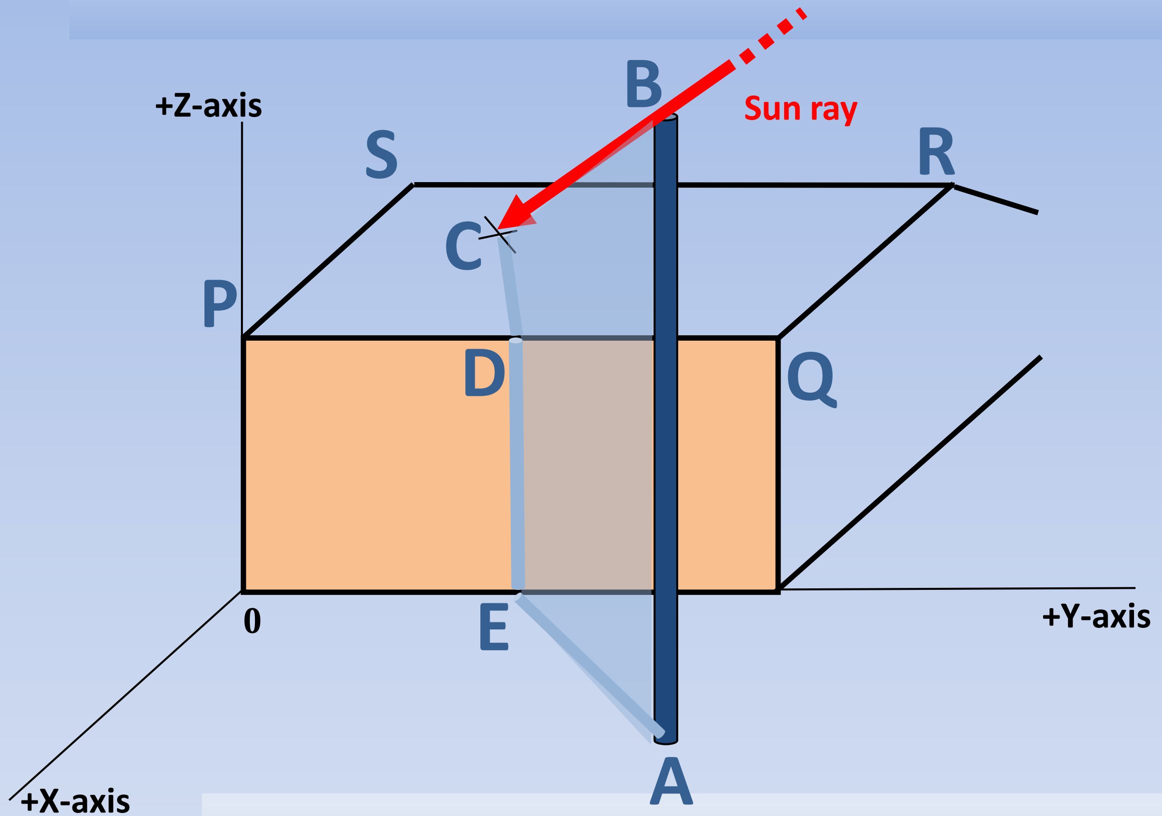
$\tau_1 = t_1 / t_2$,

$t_1 = -(x_0 \cos\xi + z_0 \sin\xi)$

$t_2 = \cos\theta \sin\beta \cos\xi + \sin\theta \sin\xi$

Special cases for $\xi = 0^0$ or 90^0.

5. Shadow lines of shading objects



Question :

Is there a shadow line on the roof?
If so, is it within the area of the panels?

Or:
Does plane PQRS (= area of panels) intersect with shadow plane ABCDE?
(Intersection line = CD).

Answer:

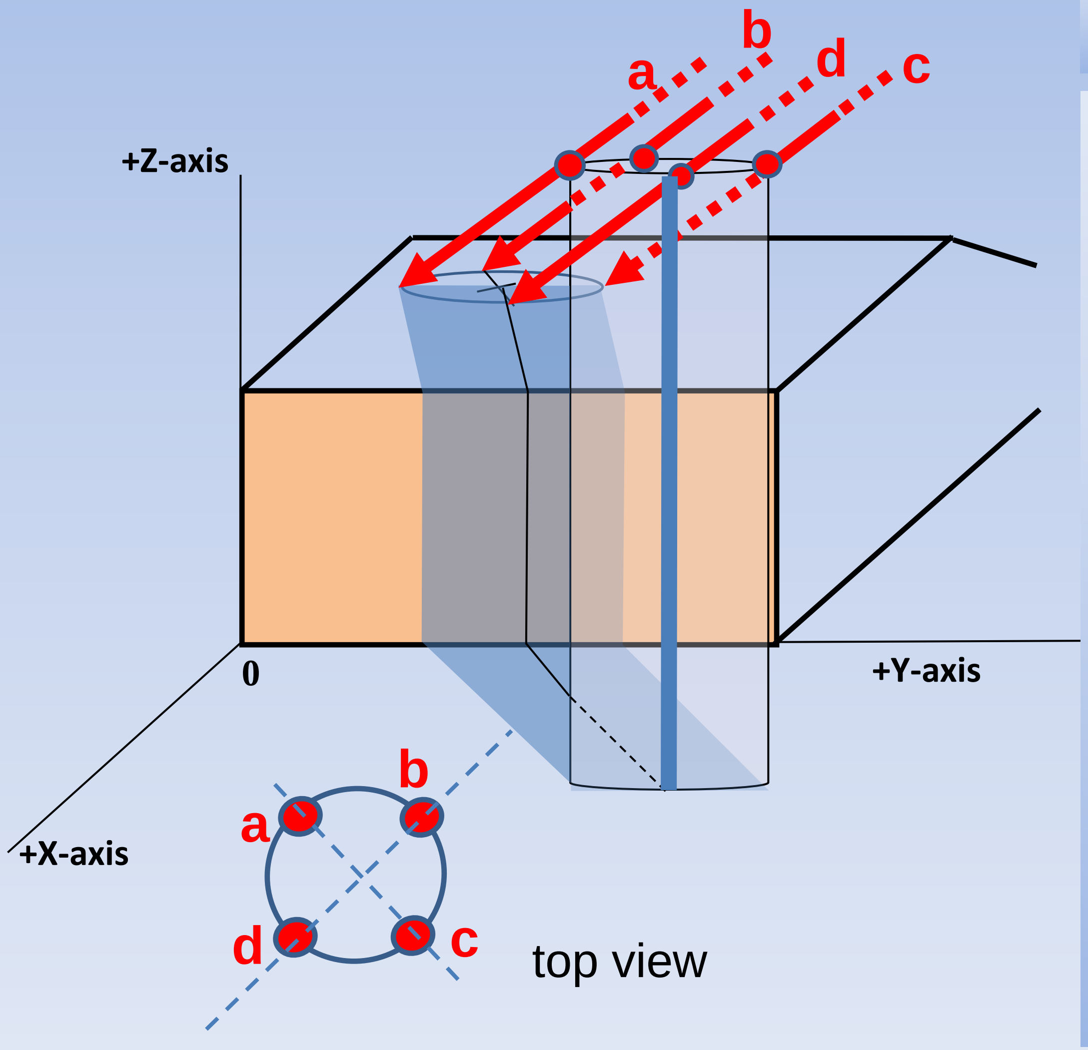
Calculate **intersection points C and D**,

Note: C and/or D might be positioned outside the area of the panels or the roof.

In those cases: calculate the intersection points of CD with the panel borders.

5. Shadow patches of shading objects

Cylindrically shaped object



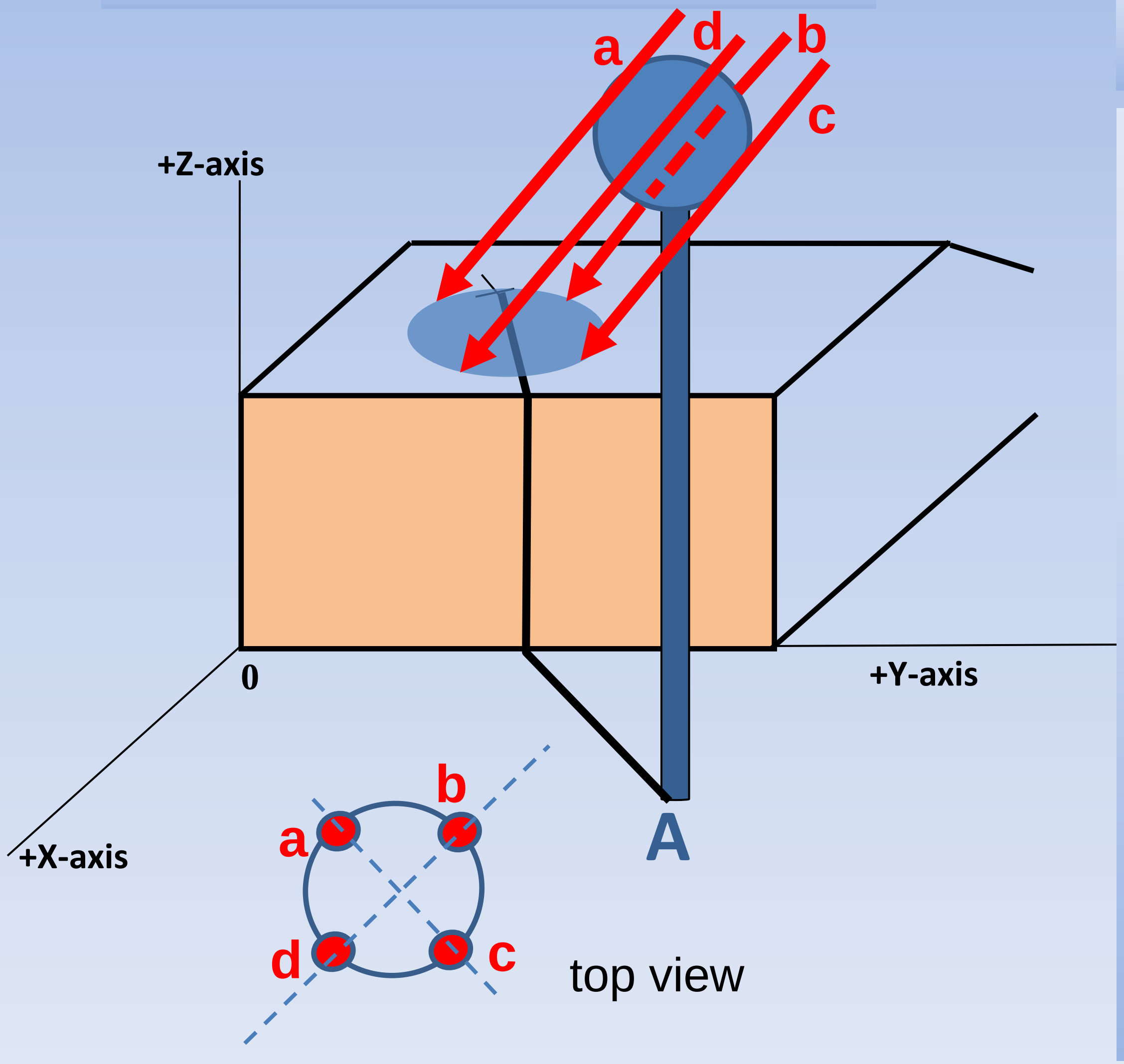
Question :
Where are the shadow patches ?

Answer:

1. Construct sun ray tracks to the roof for 4 limiting sun rays **a**, **b**, **c**, and **d**, passing the edges of the top circle.
2. Check their positions on the roof, and
3. Construct the shadow ellipse from these positions.
4. Check its position with the coordinates of the panels.
5. Construct the whole shadow patch, downward from the ellipse.

5. Shadow patches of shading objects

Spherically shaped object



top view

Question :
Where is the shadow patch ?

Answer:

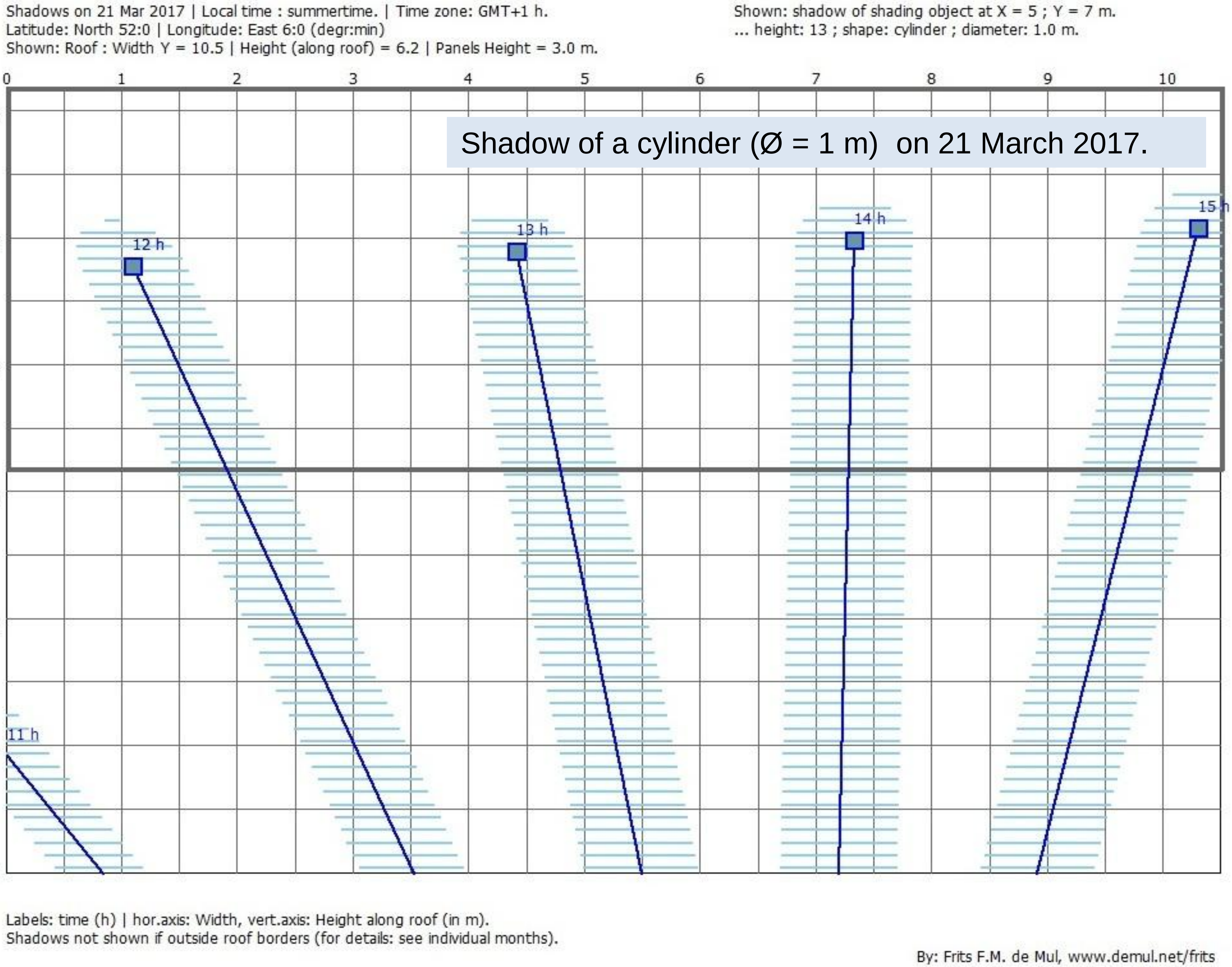
1. Construct sun ray tracks to the roof for 4 limiting sun rays **a**, **b**, **c**, and **d,** passing the edges of the cross section circle.
2. Check their positions on the roof, and
3. Construct the shadow ellipse from these positions.
4. Check its position with the coordinates of the panels.

Examples of shadow patches

(taken from the program):

1. A cylinder, diameter 1 m, on 21 March 2017
2. A sphere, diameter 2 m, on 21 March 2017
3. This sphere, on the 21st of all months of 2017

Shadow of a cylinder (Ø = 1 m) on 21 March 2017.



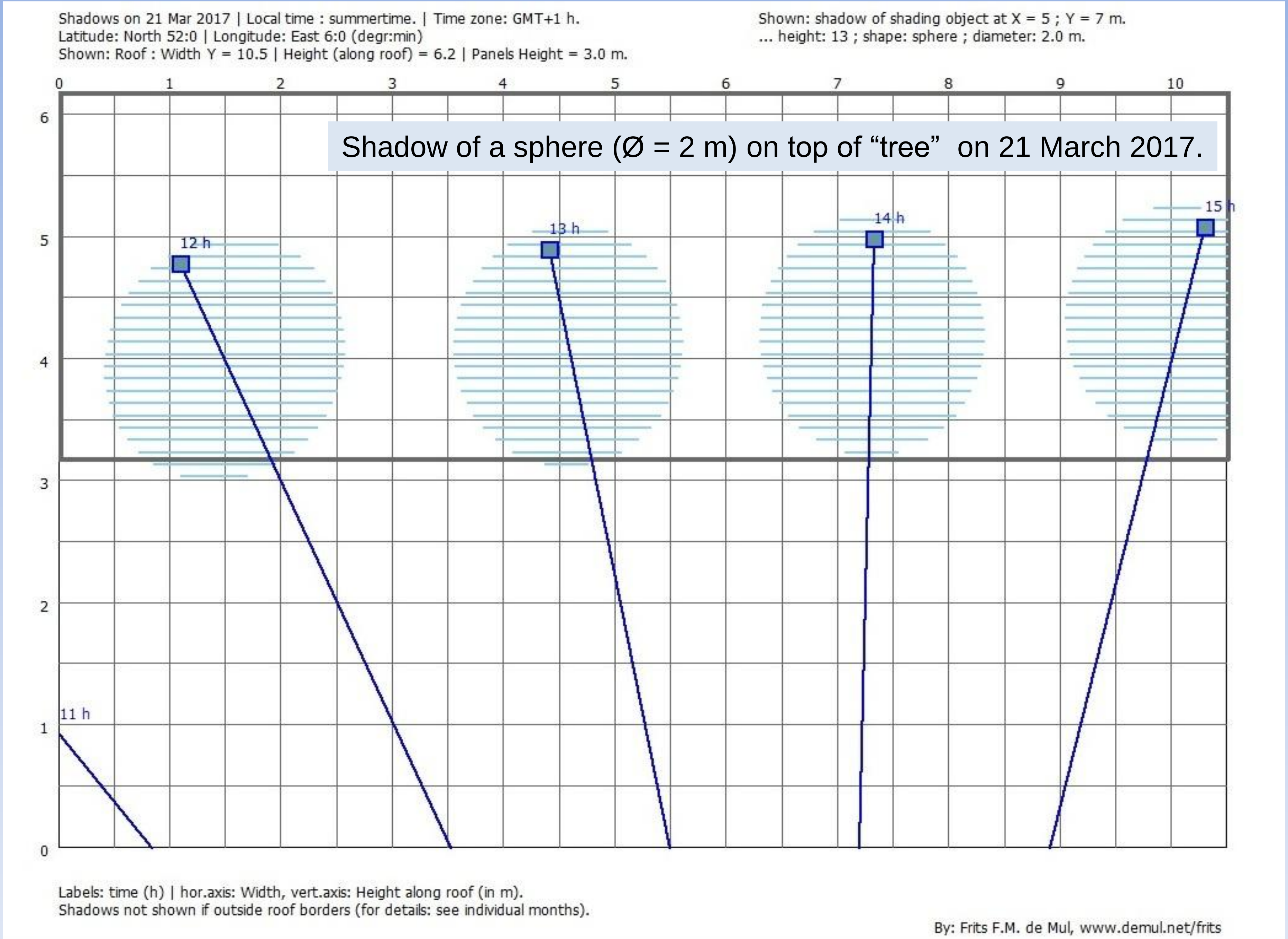
Shadows on 21 Mar 2017 | Local time : summertime. | Time zone: GMT+1 h.
Latitude: North 52:0 | Longitude: East 6:0 (degr:min)
Shown: Roof : Width Y = 10.5 | Height (along roof) = 6.2 | Panels Height = 3.0 m.

Shown: shadow of shading object at X = 5 ; Y = 7 m.
... height: 13 ; shape: cylinder ; diameter: 1.0 m.

Labels: time (h) | hor.axis: Width, vert.axis: Height along roof (in m).
Shadows not shown if outside roof borders (for details: see individual months).

By: Frits F.M. de Mul, www.demul.net/frits

Shadow of a sphere (Ø = 2 m) on top of “tree” on 21 March 2017.



Shadows on 21 Mar 2017 | Local time : summertime. | Time zone: GMT+1 h.
Latitude: North 52:0 | Longitude: East 6:0 (degr:min)
Shown: Roof : Width Y = 10.5 | Height (along roof) = 6.2 | Panels Height = 3.0 m.

Shown: shadow of shading object at X = 5 ; Y = 7 m.
... height: 13 ; shape: sphere ; diameter: 2.0 m.

Labels: time (h) | hor.axis: Width, vert.axis: Height along roof (in m).
Shadows not shown if outside roof borders (for details: see individual months).

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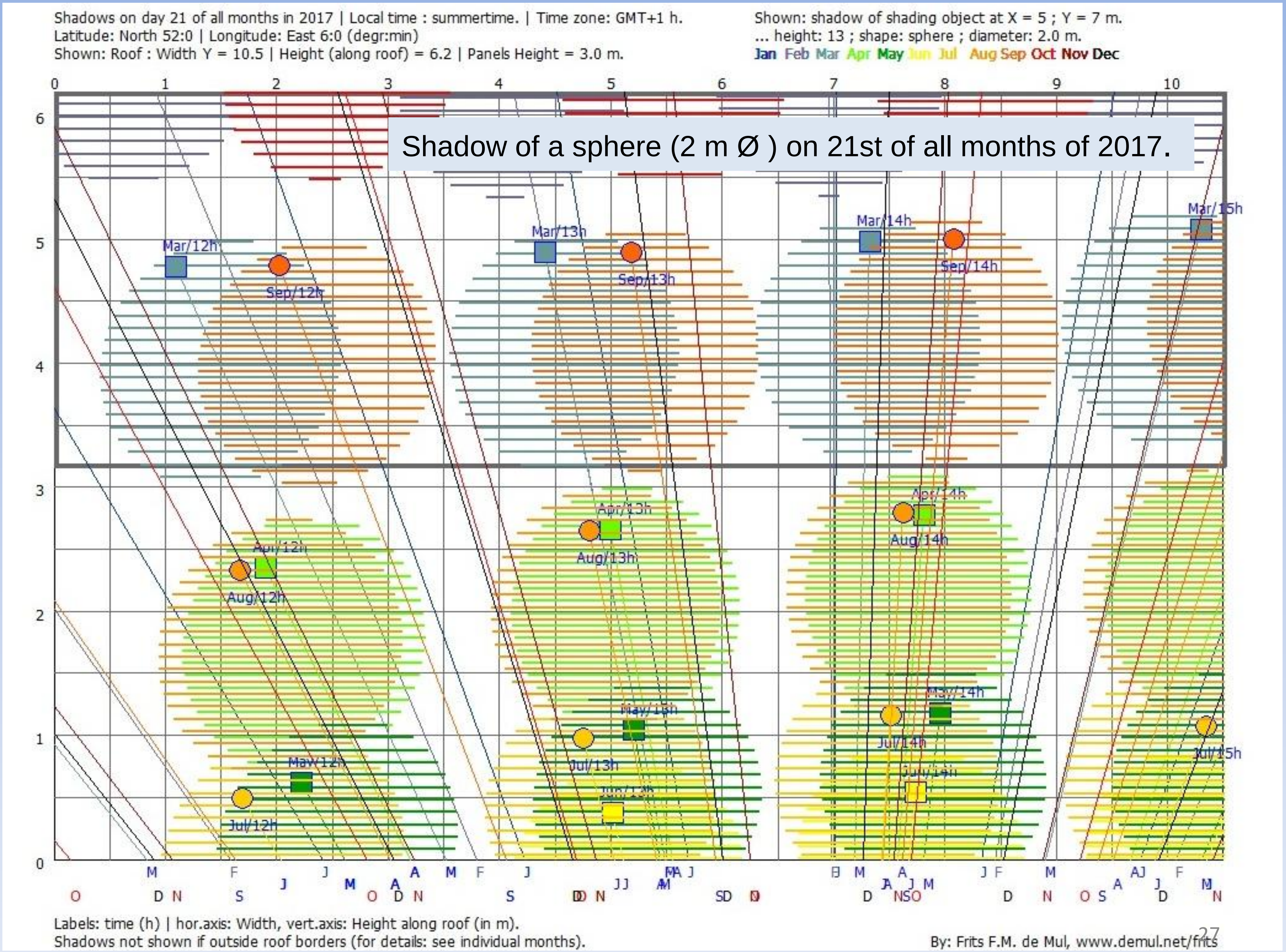
Shadow of a sphere (2 m Ø) on 21st of all months of 2017.



Shadows on day 21 of all months in 2017 | Local time : summertime. | Time zone: GMT+1 h.
Latitude: North 52:0 | Longitude: East 6:0 (degr:min)
Shown: Roof : Width Y = 10.5 | Height (along roof) = 6.2 | Panels Height = 3.0 m.

Shown: shadow of shading object at X = 5 ; Y = 7 m.
... height: 13 ; shape: sphere ; diameter: 2.0 m.

Labels: time (h) | hor.axis: Width, vert.axis: Height along roof (in m).
Shadows not shown if outside roof borders (for details: see individual months).

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6. Atmospheric effects on energy efficiency

Corrections:

1. Atmospheric transmission
   due to attenuation (absorption, scattering …)

2. Cloudy skies

3. Atmospheric refraction

6. Atmospheric effects: air transmission

Correction *CF* of *E* for transmission through atmosphere

(due to scattering and absorption) for clear sky (ref. Guttman):

$$\boldsymbol{CF = \exp(-\mu . w)}$$

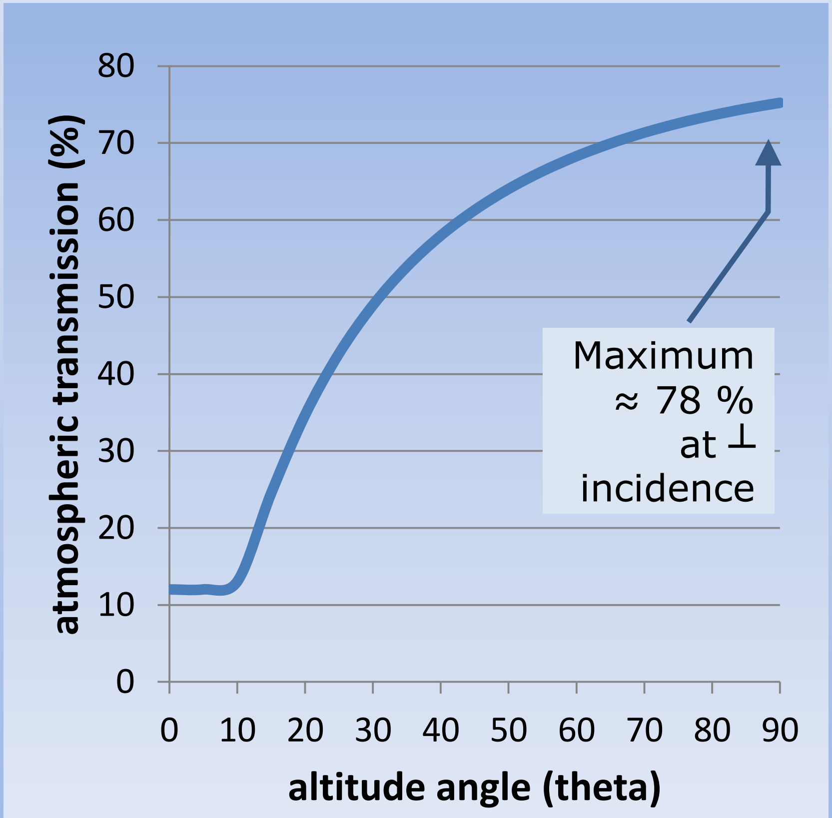
μ = attenuation coefficient,
in 1/"air-mass" (≈ 0.25)
w = amount of "air-mass"; approx.:

$$\boldsymbol{w = 1/\sin\theta} \quad \text{if } \theta >= 10°$$
$$= 1/\sin 10° \quad \text{if } \theta < 10°$$

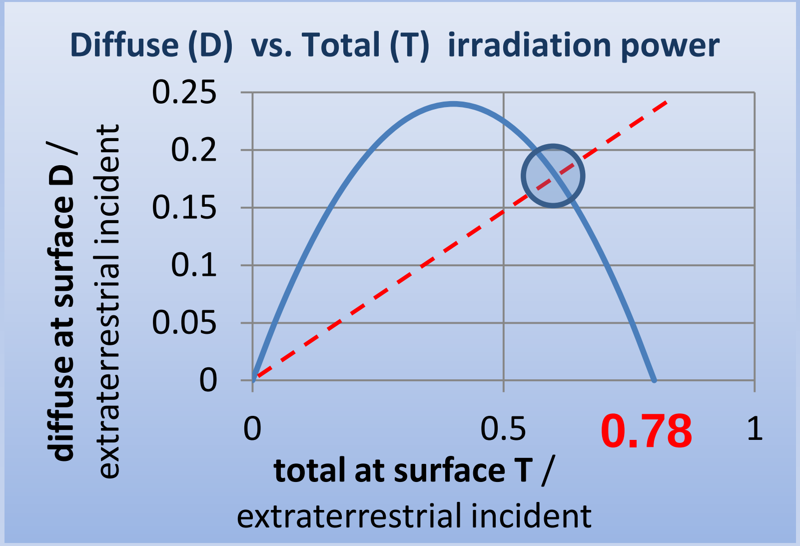
(θ = altitude (elevation) angle)

Relative correction factor
w.r.t. perpendicular incidence ($\theta = 90^0$):

$$CF = \exp\{-\mu.(w-1)\}$$



6. Atmospheric effects: clear and cloudy skies.

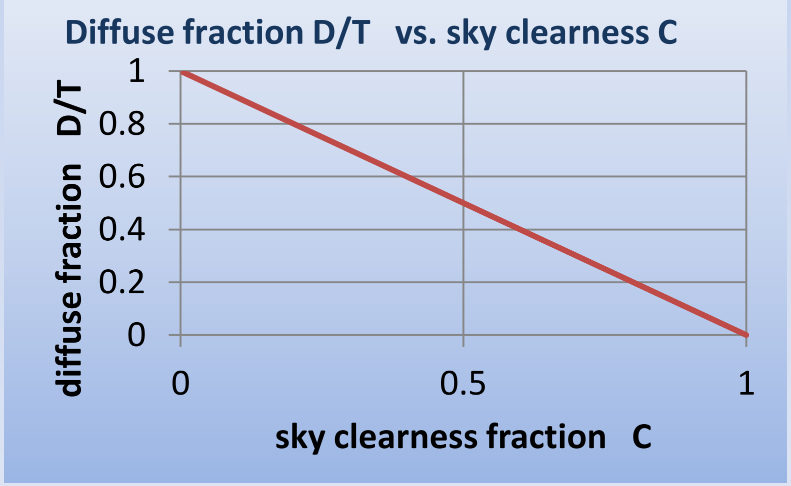


Diffuse (D) vs. total (T = direct + diffuse**) irradiation power** measurements,

for horizontal surface (**zenith angle = 0°**), averaged over day and year: (Liu & Jordan).

Experimental errors and band widths ± ≈ 25 %.

NB. For other zenith angles, or other latitudes, minor deviations.

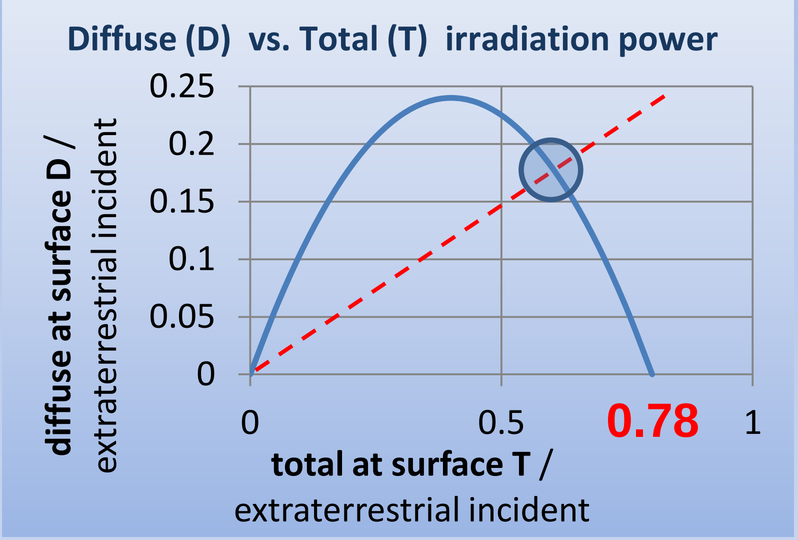


Approach:

1. Select clearness C.
2. Determine D/T from clearness C:

- **D = (1 – C) T** (dashed line in upper fig.)
- Determine intersection point in upper fig.
- This results in **D / T .**

6. Atmospheric effects: clear and cloudy skies.



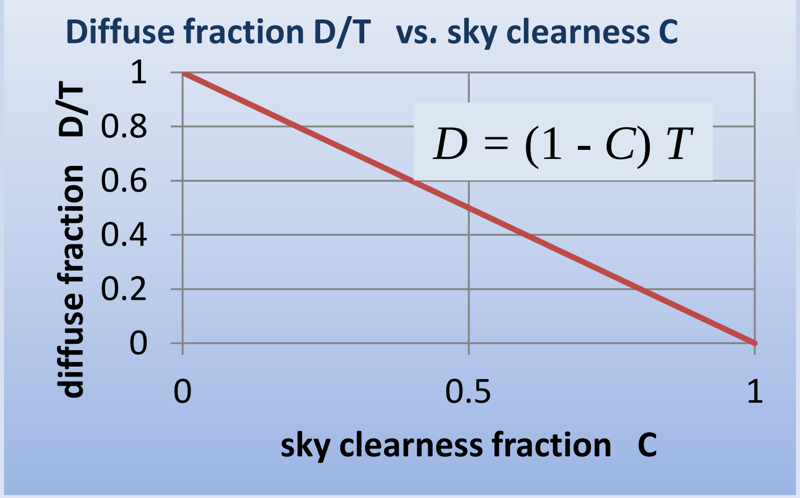
Diffuse (D) vs. total (T) irradiation power (total = direct + diffuse) for horizontal surface (zenith angle = 0°), averaged over day and year:

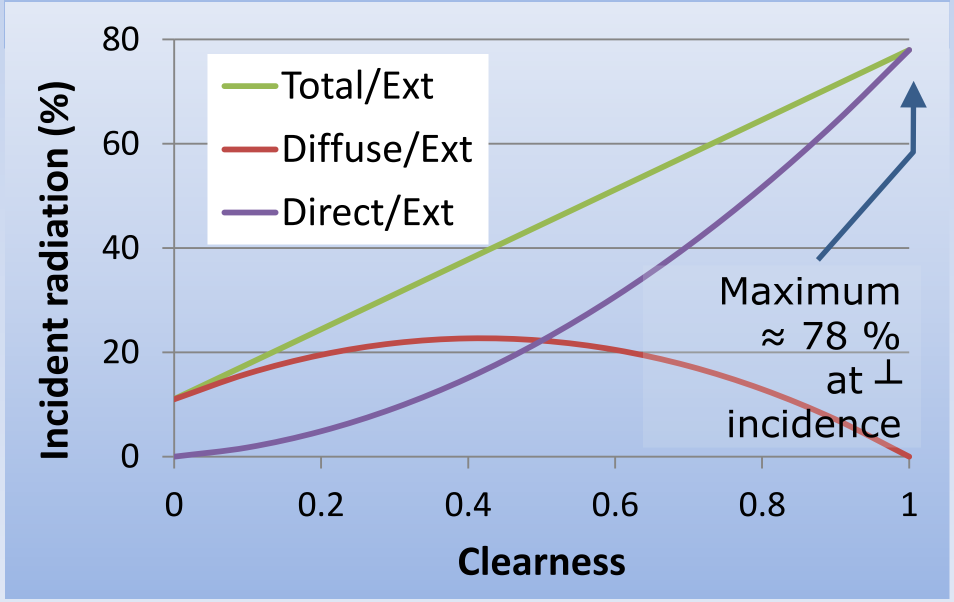
Derived from Liu & Jordan:

$$\frac{D}{Ext} = 0.78(1-C) - 0.67(1-C)^2$$

(ext = extraterrestrial incident from space)

$$T = D/(1-C)$$

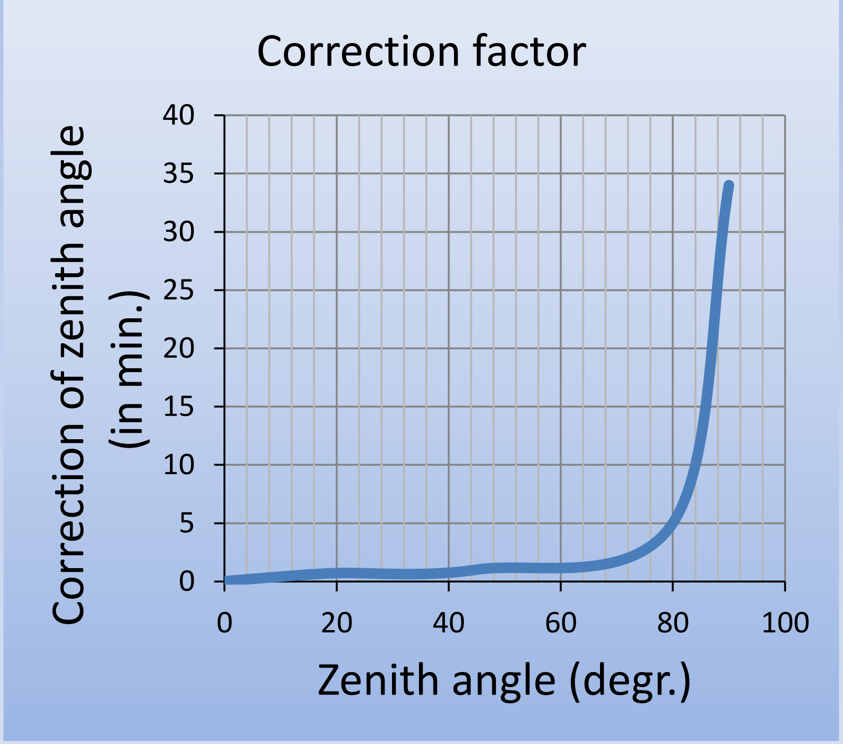




6. Atmospheric effects: refraction

Correction for atmospheric refraction:

due to changes in refraction coefficient



Effect at low altitudes only.

For energy efficiency calculations this effect is small and will be ignored.

Zenith angle = 90^0 - altitude (elevation) angle

7. Energy output (in kWh)

Energy output **(in kWh):** by numerical integration over time intervals

1. Calculate for each time interval:

   Irradiation efficiency, corrected for **air clearness, transmission** and **refraction**, (%)

2. Multiply with chosen **time interval duration** (h)

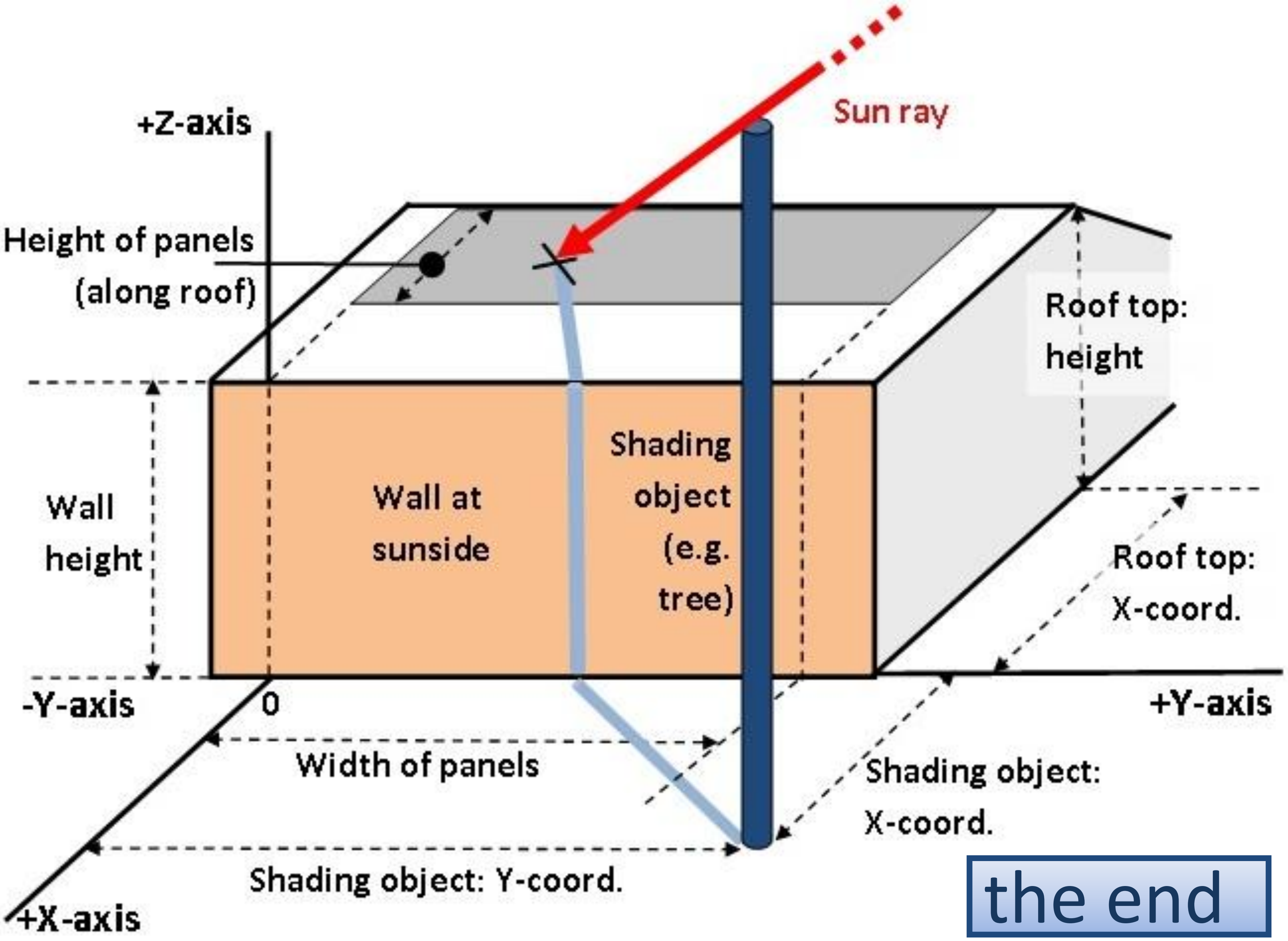
3. Sum over **all intervals** (per day, month or year)

4. Multiply the result with:

   - **Solar irradiance power** (normally 1300-1400 W/m^2)
   - **Intrinsic (electronic) efficiency** of the solar panels (%)
   - **Exposed area** of the solar panels (m^2)

→ **Computer program**:

- calculation for clear sky, and
- corrections for shadows and atmospheric effects..


Sun ray
+Z-axis
Height of panels
(along roof)
Roof top:
height
Shading
object
(e.g.
tree)
Wall
height
Wall at
sunside
Roof top:
X-coord.
-Y-axis
0
+Y-axis
Width of panels
Shading object:
X-coord.
Shading object: Y-coord.
+X-axis

the end