Electric Field of a Long Wire

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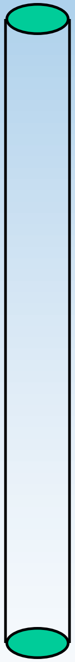
Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

E-field of a long wire



Available:

A thin wire, infinitely long,

carrying charge density λ [C/m]

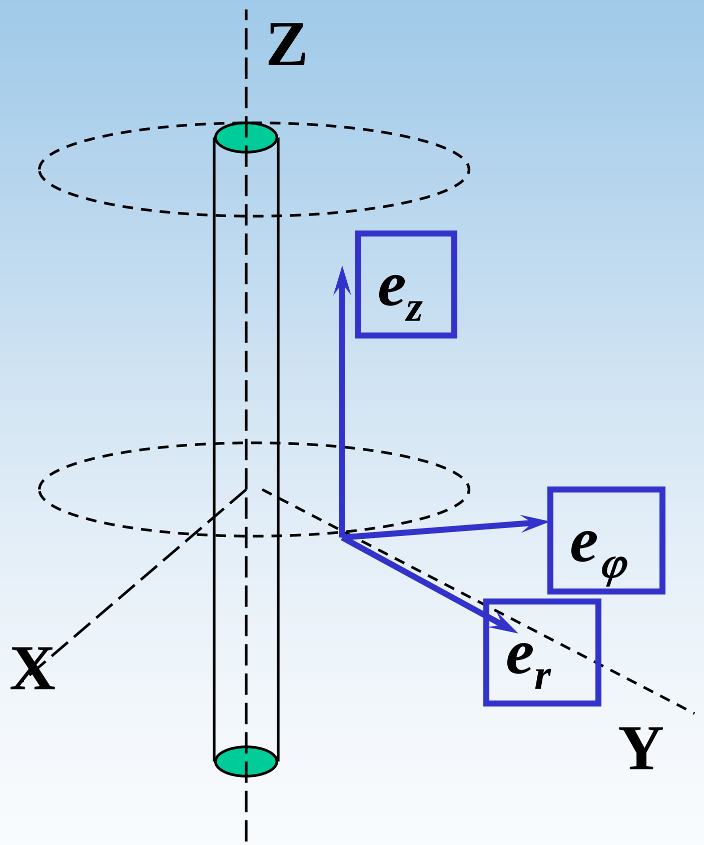
Question:

Calculate ***E***-field in arbitrary points outside the wire.

***E*-field of a long wire**

- Analysis and symmetry
- Approach to solution
- Calculations
- Conclusions
- Appendix: angular integration

Analysis and Symmetry



1. Charge distribution:

   λ [C/m]

2. Coordinate axes:

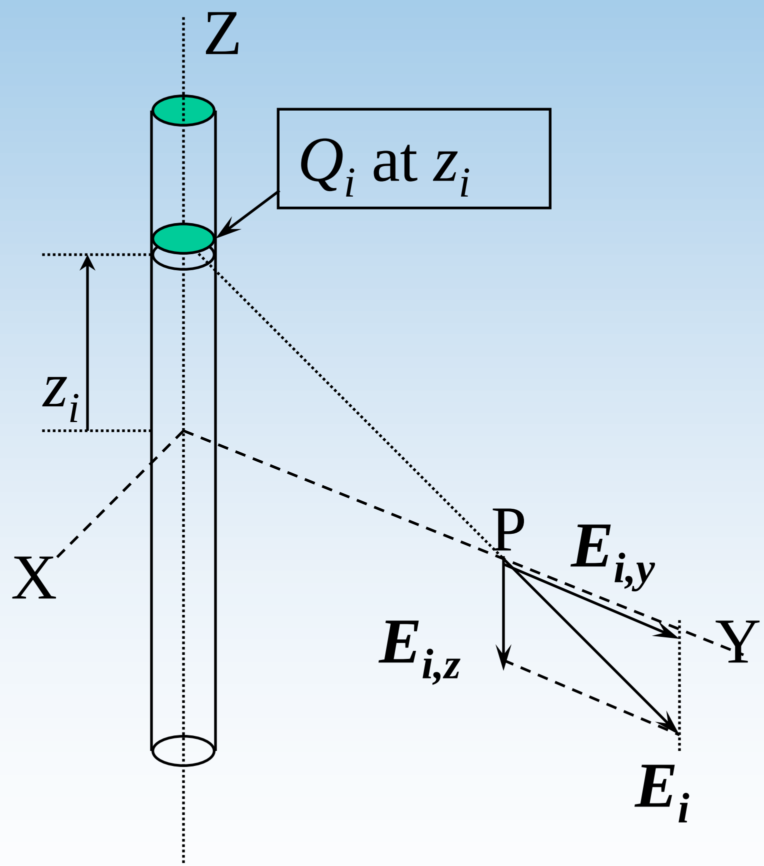
   Z-axis // wire

3. Symmetry: cylinder

4. Cylinder coordinates:

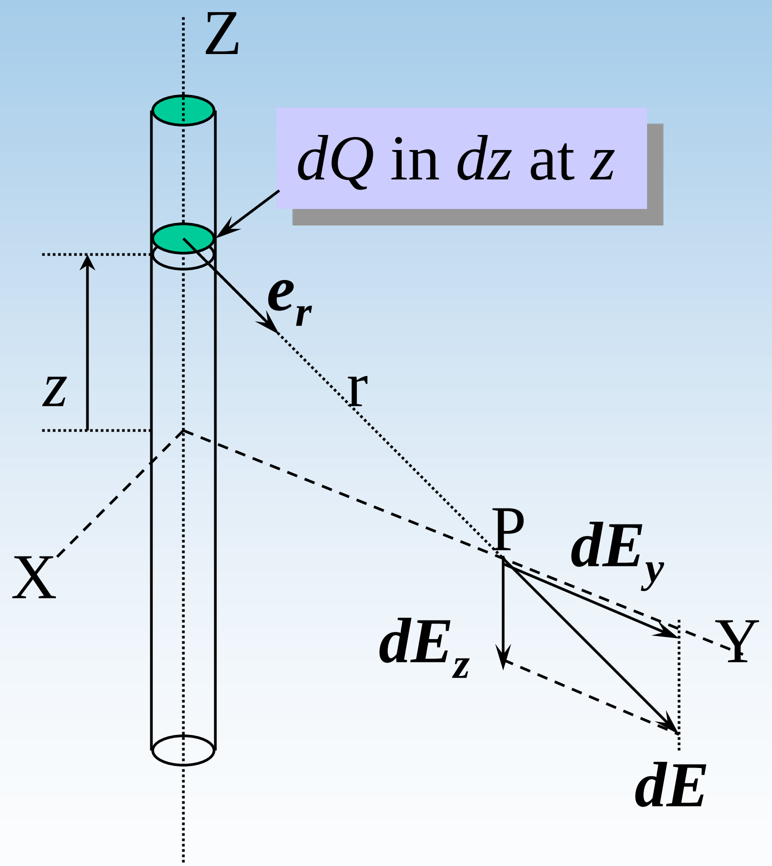
   ***r*, *z*, φ**

Analysis, field build-up



1. Coordinate axes: XYZ
2. Assume point P on Y-axis
3. all Q_i's at z_i contribute $\boldsymbol{E_i}$ to $\boldsymbol{E}$ in P
4. components $\boldsymbol{E_{i,x}}$,$\boldsymbol{E_{i,y}}$, $\boldsymbol{E_{i,z}}$
5. $\boldsymbol{E_i}$ in YZ-plane → all $\boldsymbol{E_{i,x}} = 0$!!
6. expect: $\boldsymbol{E_z} = \Sigma\, \boldsymbol{E_{i,z}} = 0$; has to be checked !!

Approach to solution



1. Distributed charges dQ

2. $$\boldsymbol{dE} = \frac{dQ}{4\pi\varepsilon_0 r^2}\boldsymbol{e_r}$$

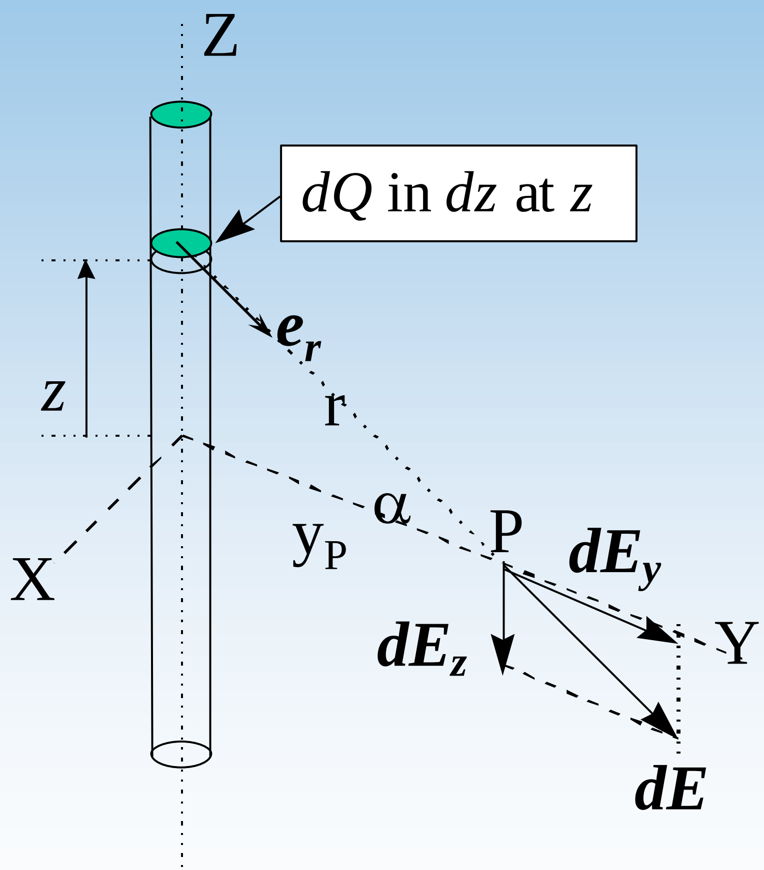
3. charge element $dQ = \lambda.dz$

4. y- and z- components

5. $$\boldsymbol{dE_y} = \frac{dQ}{4\pi\varepsilon_0 r^2}(\boldsymbol{e_r} \bullet \boldsymbol{e_y})$$

$$\boldsymbol{dE}_z = \frac{dQ}{4\pi\varepsilon_0 r^2}(\boldsymbol{e_r} \bullet \boldsymbol{e}_z)$$

Calculations (1)



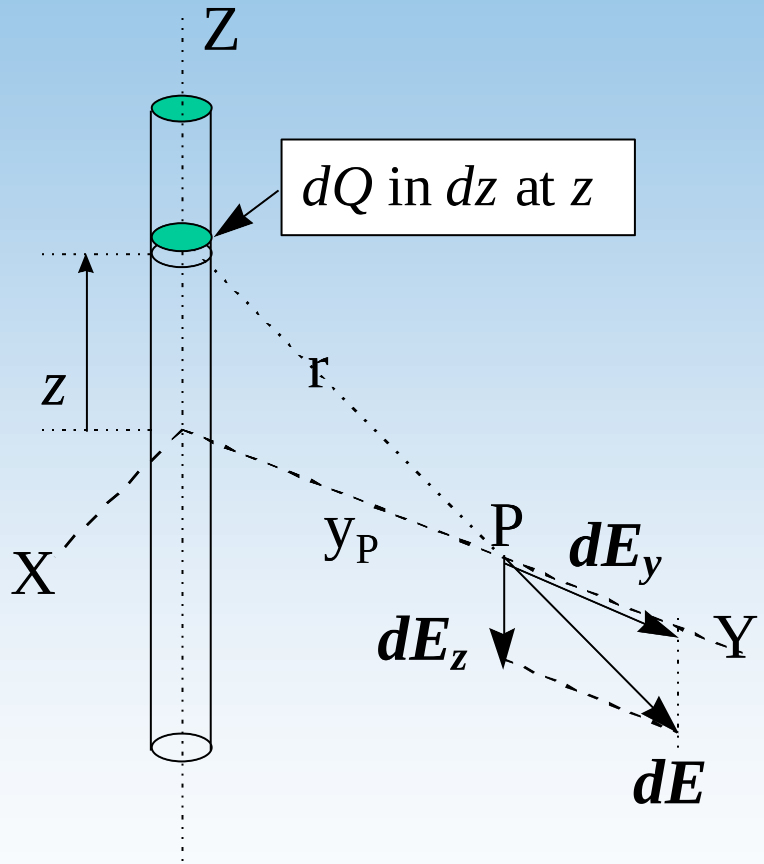
$$dE_y = \frac{\lambda dz}{4\pi\varepsilon_0 r^2}\,\boldsymbol{e_r} \bullet \boldsymbol{e_y}$$

$$\boldsymbol{e_r} \bullet \boldsymbol{e_y} = \cos\alpha = \frac{y_P}{\sqrt{z^2 + y_P{}^2}}$$

$$dE_y = \frac{\lambda dz}{4\pi\varepsilon_0\left(z^2 + y_P{}^2\right)}\frac{y_P}{\sqrt{z^2 + y_P{}^2}}$$

$$E_y = \int_{z=-\infty}^{\infty} dE_y \quad = \frac{\lambda}{2\pi\varepsilon_0 y_P}$$

Calculations (2)

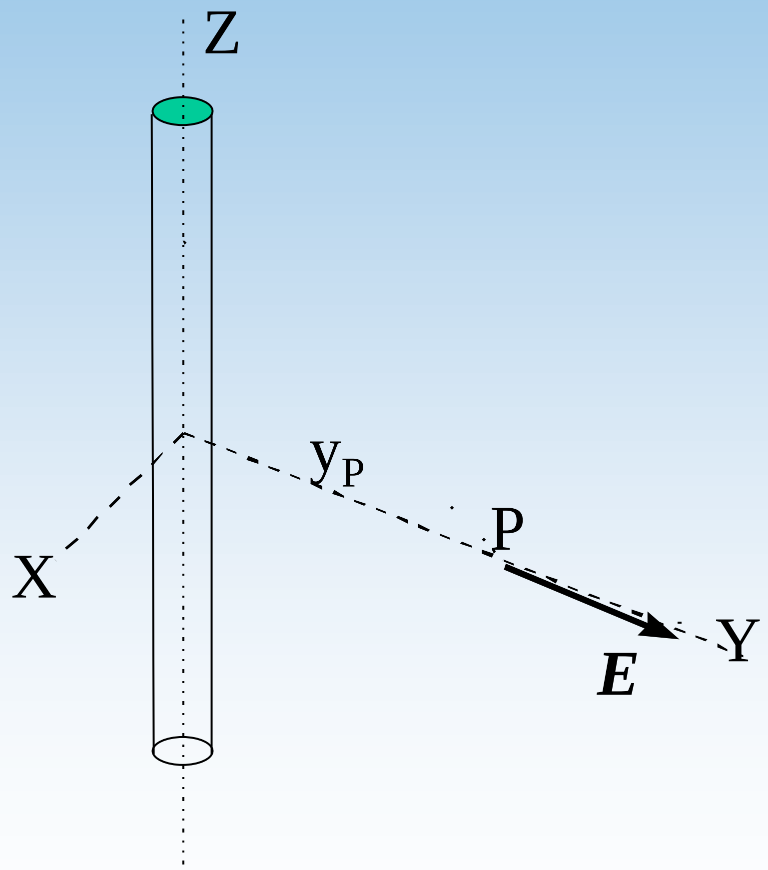


$$E_y = \int_{-\infty}^{\infty} dE_y = \frac{\lambda}{2\pi\varepsilon_0 y_P}$$

a similar calculation:

$$E_z = \int_{-\infty}^{\infty} dE_z = 0$$

Conclusions

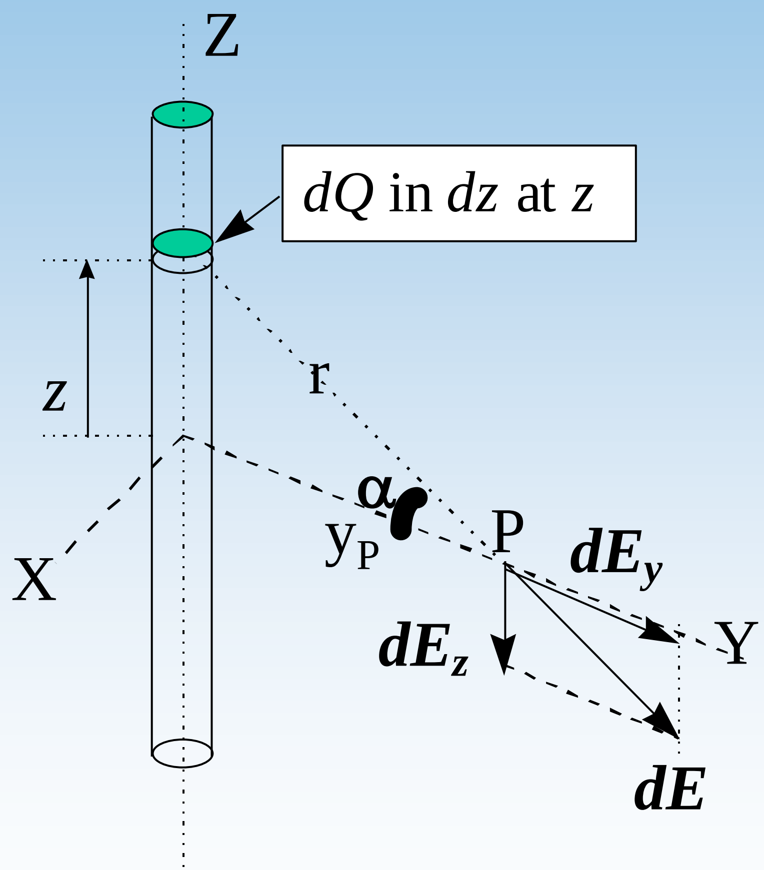


$$\boldsymbol{E} = \frac{\lambda}{2\pi\varepsilon_0 y_P}\boldsymbol{e_y}$$

$|\boldsymbol{E}| \sim 1/y_P$

cylinder symmetry

Appendix: angular integration (1)



$$dE_y = \frac{\lambda dz}{4\pi\varepsilon_0 r^2}\cos\alpha$$

z, *r* and *f* (*α*) → write *z*, *r* = *f* (*α*) and integrate over *α*.

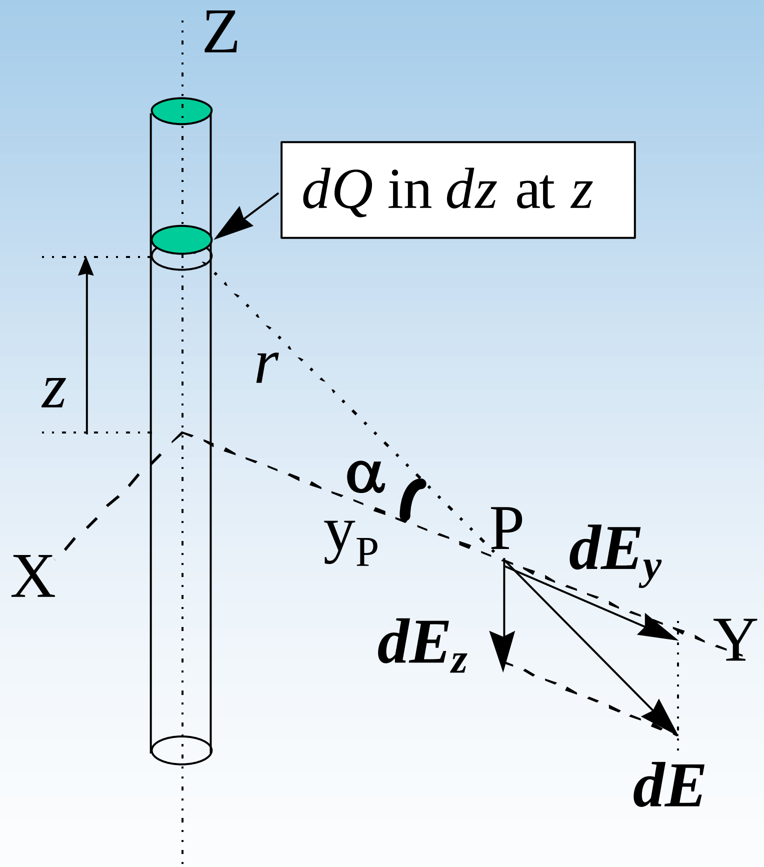
$$r = f(\alpha) = \frac{y_P}{\cos\alpha}$$

$$z = y_P \tan\alpha \qquad dz = \frac{y_P d\alpha}{\cos^2\alpha}$$

$$dE_y = \frac{\lambda y_P \cos^{-2}\alpha d\alpha}{4\pi\varepsilon_0 y_P^2 \cos^{-2}\alpha}\cos\alpha$$

$$dE_y = \frac{\lambda}{4\pi\varepsilon_0 y_P}\cos\alpha d\alpha$$

Appendix: angular integration (2)



$$dE_y = \frac{\lambda}{4\pi\varepsilon_0 y_P}\cos\alpha d\alpha$$

integration over α

from -π/2 to π/2 :

$$\boxed{E_y = \frac{\lambda}{2\pi\varepsilon_0 y_P}}$$

Cylindrical symmetry

the end