Electric field of a Polarized Object

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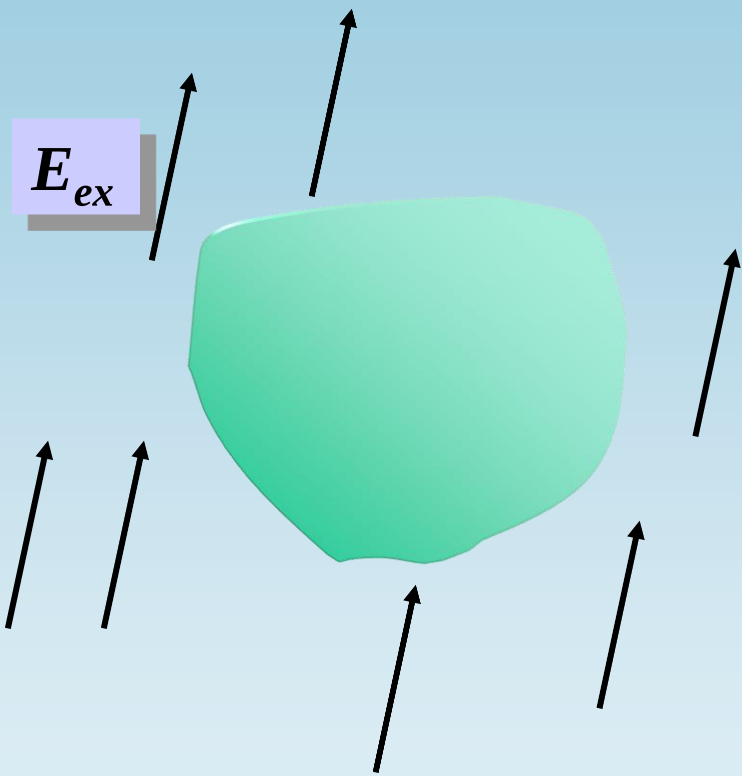
Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

Electric field of a Polarized Object



<u>Available:</u>

An external ***E***-field: $\boldsymbol{E}_{ex}$.

A dielectric object will become polarized.

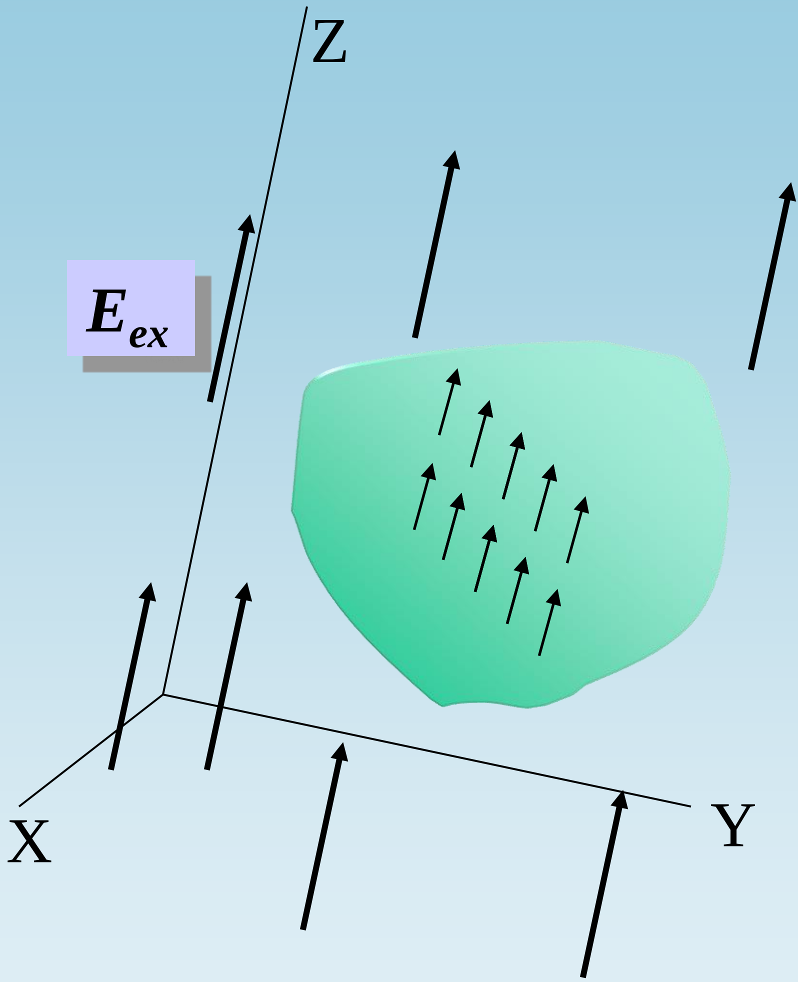
<u>Question:</u>

Calculate ***E***-field produced BY (not: IN) the polarized object.

Electric field of a Polarized Object

- Analysis and symmetry
- Approach to solution
- Calculations
- Conclusions

Analysis and Symmetry



Coordinate axes:

assume Z-axis // $\boldsymbol{E}_{ex}$

Result of polarization:

Dipole distribution:

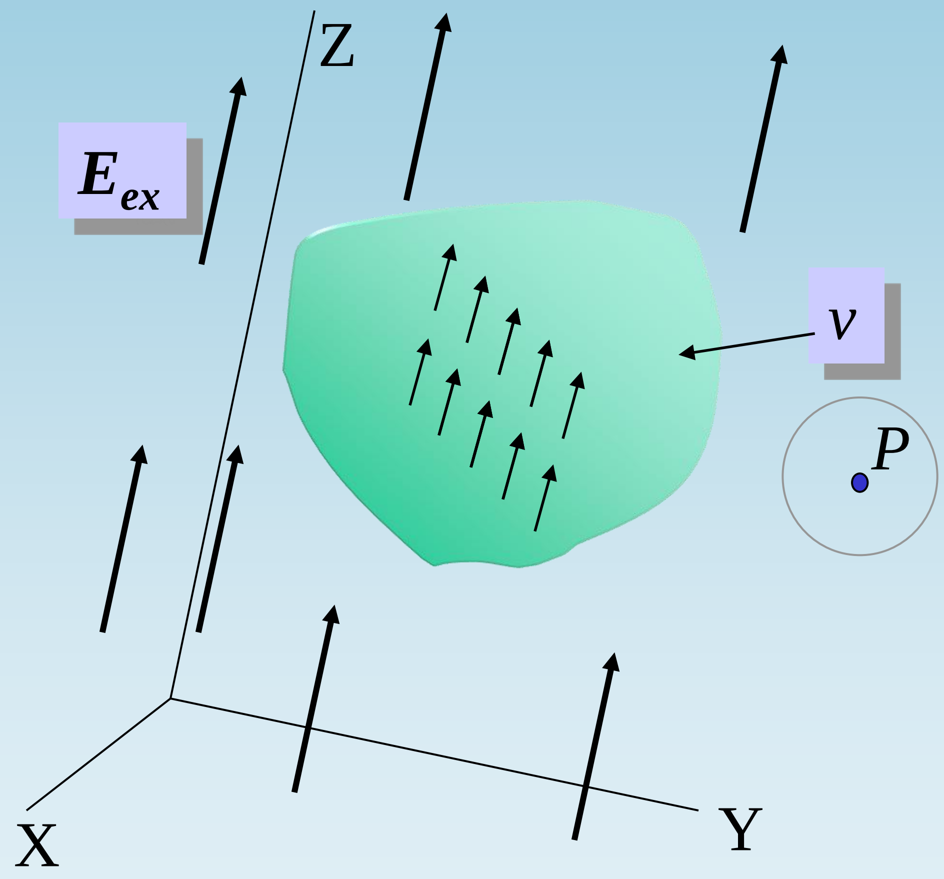
n dipoles/m^3 ;

each dipole moment $\boldsymbol{p}$ [Cm]

Dipoles will be directed along $\boldsymbol{E}_{ex}$: $\boldsymbol{p}$ // $\boldsymbol{E}_{ex}$

n and $\boldsymbol{p}$ homogeneous

Approach to solution



<u>Question:</u> calculate ***E***-field in arbitrary point *P* outside *v*

<u>Approach:</u>

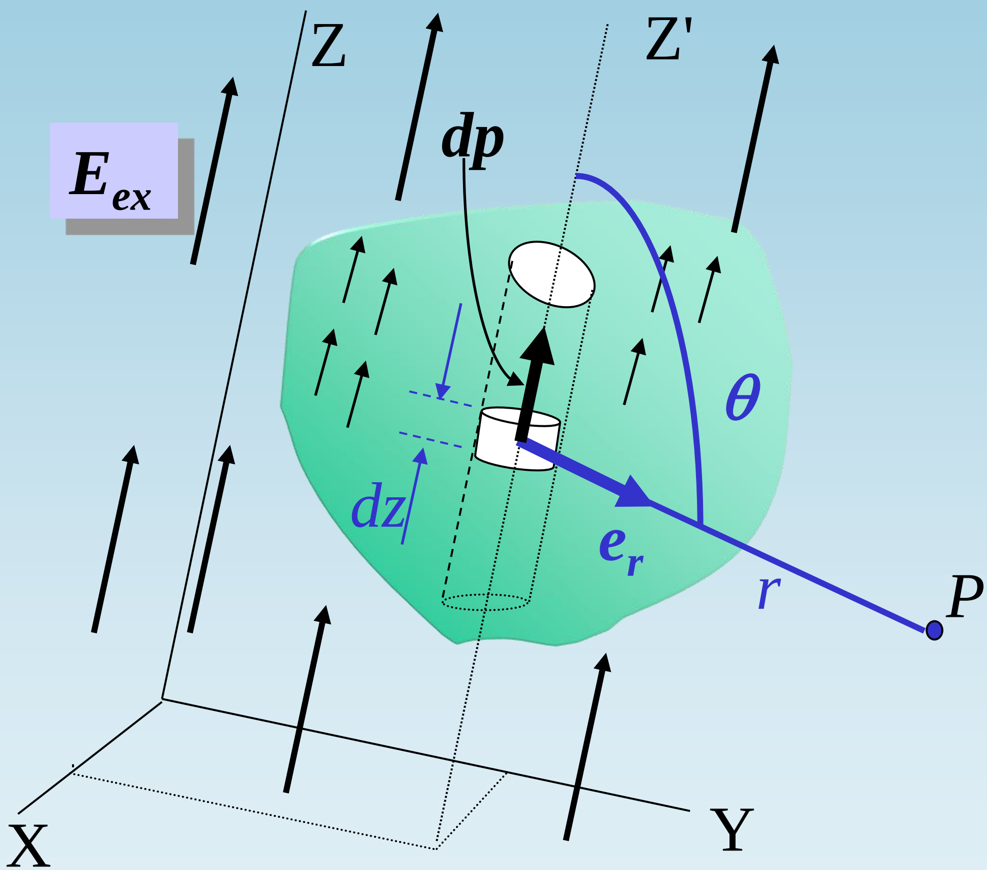
= calculate potential *V* ;

= ***E*** from *V* by differentiation

<u>Distributed dipoles:</u>

dV- integration over volume elements *dv* filled with ***dp*.**

Calculations (1)



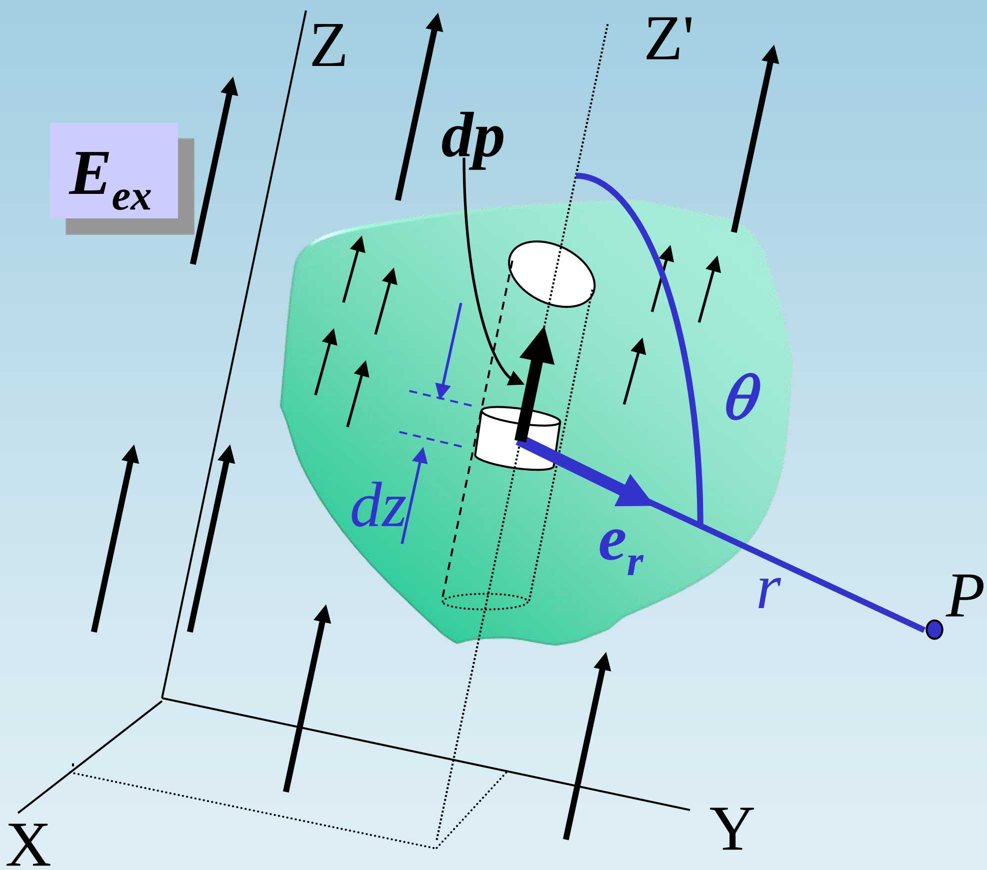
volume element dv with $dp = np.dv = np.dS_{\perp}.dz$; with $dS_{\perp} \perp$ Z-axis

Dipole potential: $$V = \frac{\mathbf{p} \bullet e_r}{4\pi\varepsilon_0 r^2}$$

$$dV = \frac{dp \cos\theta}{4\pi\varepsilon_0 r^2}$$

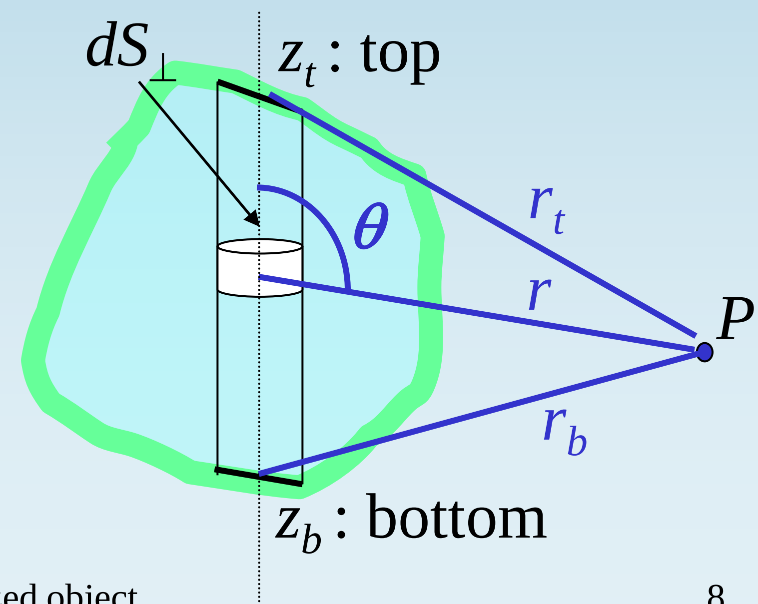
$$= \frac{np.dS_{\perp} dz.\cos\theta}{4\pi\varepsilon_0 r^2}$$

Calculations (2)

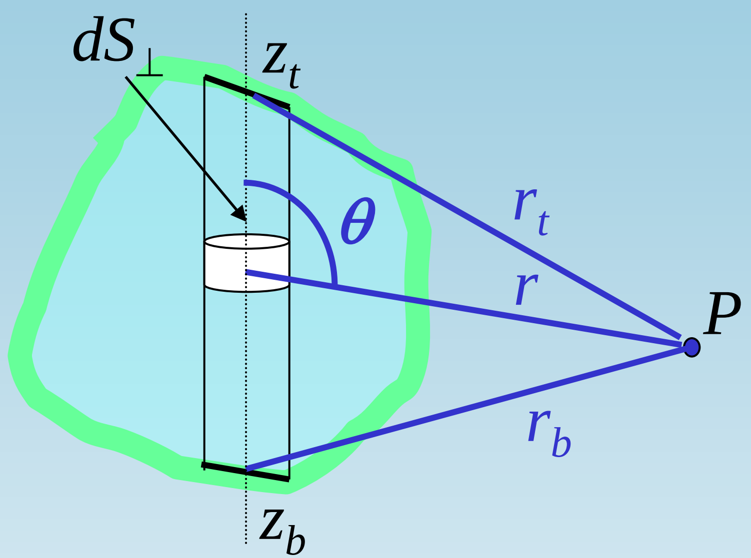


$$dV \; \frac{np.dS_{\perp}.dz}{4\pi\varepsilon_0 r^2}\cos\theta$$

cross section through *P* and Z'-axis :



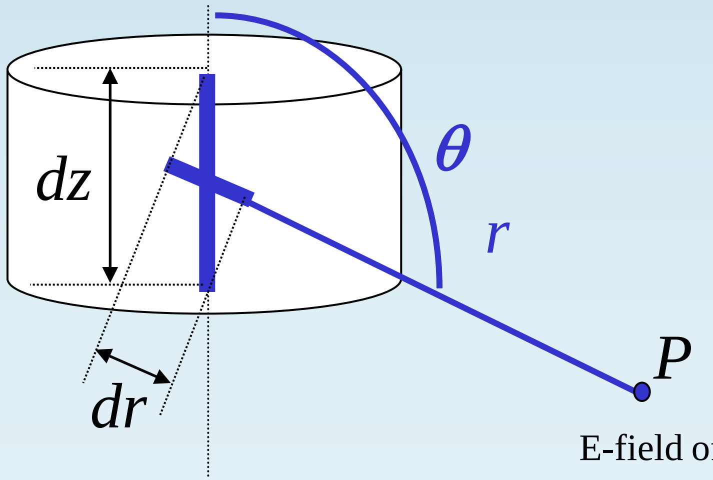
Calculations (3)



$$dV = \frac{np.dS_\perp.dz}{4\pi\varepsilon_0 r^2}\cos\theta$$

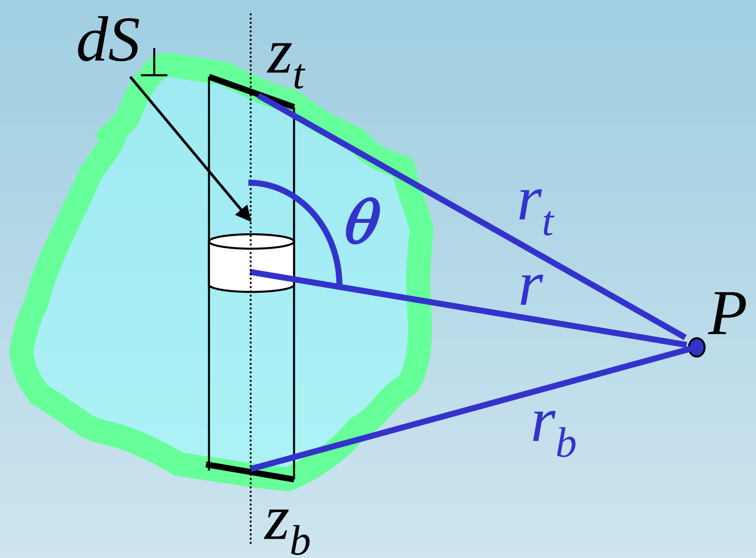
dz-integration ⇒
dr-integration :

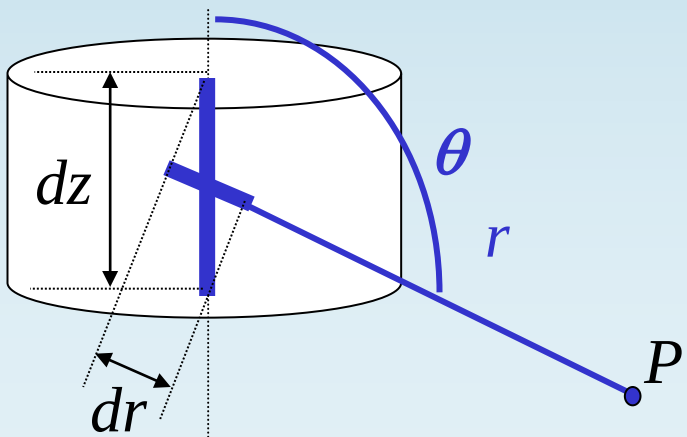
$dr = dz.\cos\theta \Rightarrow dz = dr/\cos\theta$



$$dV = \frac{np.dS_\perp.dr}{4\pi\varepsilon_0 r^2}$$

Calculations (4)





$$dV = \frac{np.dS_{\perp}.dr}{4\pi\varepsilon_0 r^2}$$

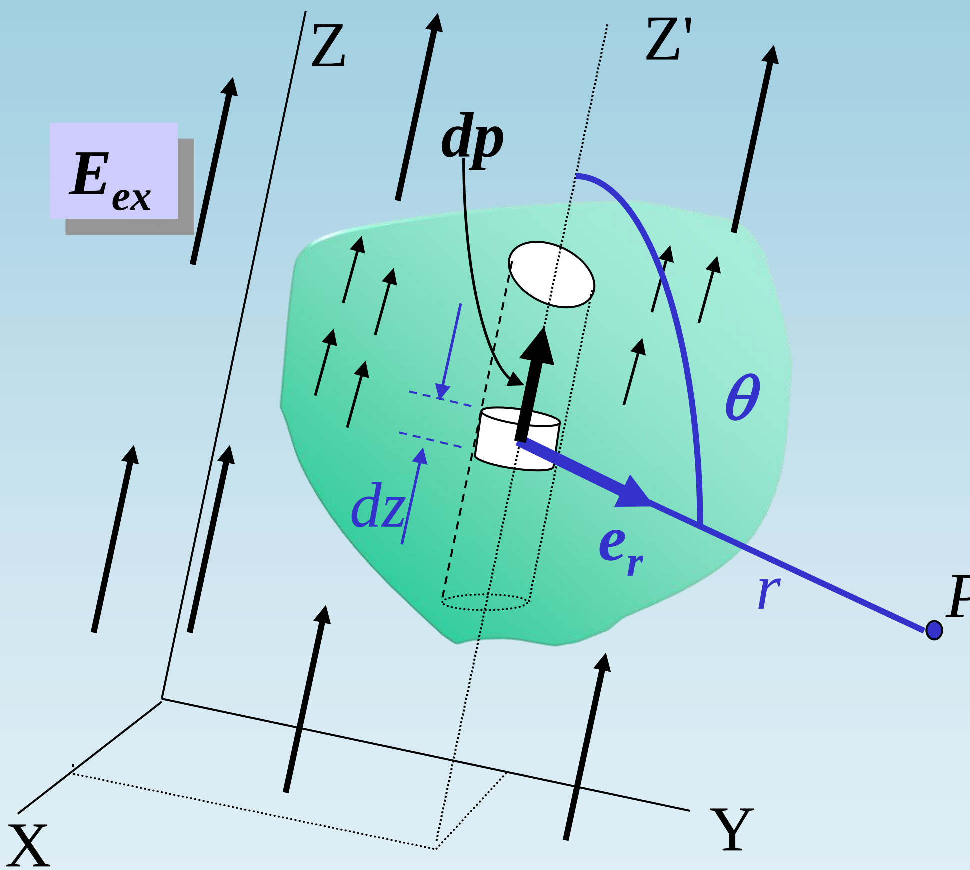
$$V = \iint_S dS_{\perp} \int_{r_b}^{r_t} \frac{np.dr}{4\pi\varepsilon_0 r^2}$$

Def.: "**Polarization**" $\boldsymbol{P} = n\boldsymbol{p}$

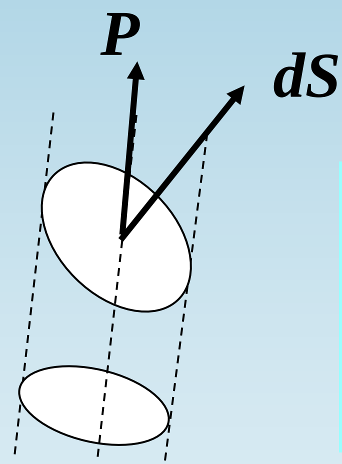
$\boldsymbol{P}$ in [m^{-3} .Cm] = [C/m^2]

$$V = \iint_S dS_{\perp} \frac{P}{4\pi\varepsilon_0}\left[\frac{1}{r_t} - \frac{1}{r_b}\right]$$

Calculations (5)



$$V = \iint_S dS_\perp \frac{P}{4\pi\varepsilon_0}\left[\frac{1}{r_t} - \frac{1}{r_b}\right]$$



$$P.dS_\perp = \boldsymbol{P.dS}$$

bound surface charges:

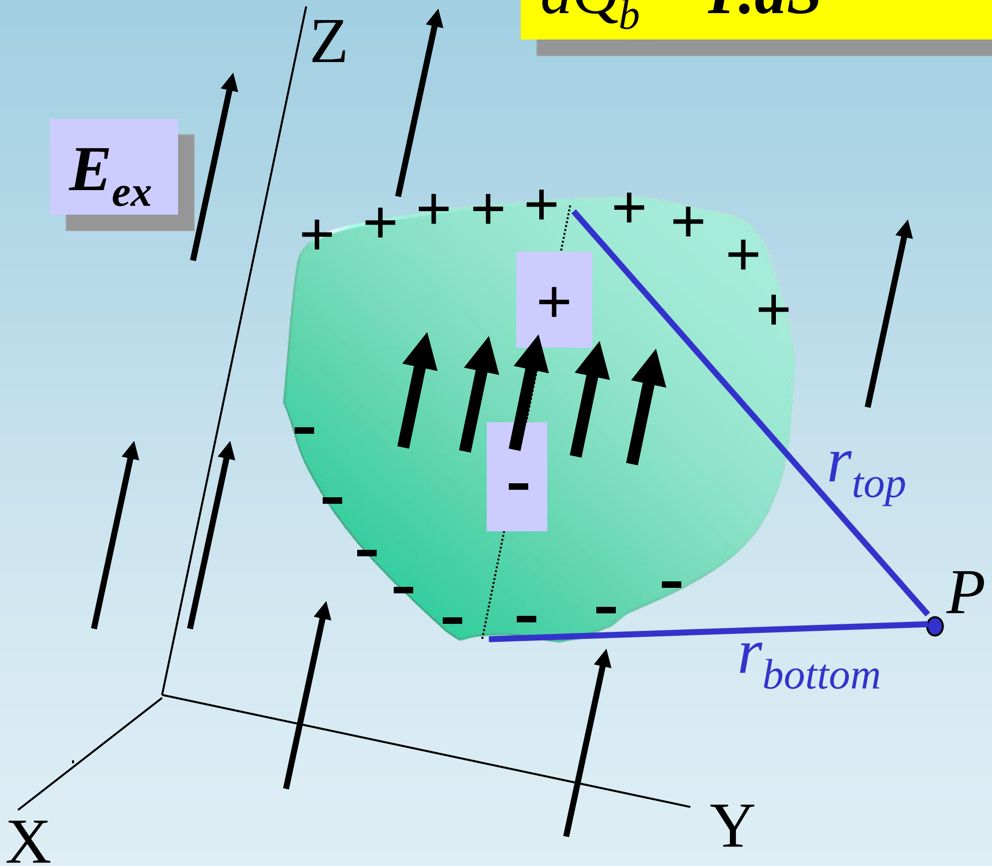
$$dQ_b = \boldsymbol{P.dS}$$

$$V = \iint_S \frac{dQ_b}{4\pi\varepsilon_0}\left[\frac{1}{r_t} - \frac{1}{r_b}\right]$$

Conclusions

bound surface charges:
dQ_b = ***P.dS***

$$V = \iint_S \frac{dQ_b}{4\pi\varepsilon_0}\left[\frac{1}{r_t} - \frac{1}{r_b}\right]$$



<u>Conclusion:</u>

the field of a **polarized volume** is equivalent to

the field of **bound surface charges** (provided homogeneous polarization).

"Polarization" = bound surface charge density [C.m^{-2}]

the end