Electric dipoles on a thin long line

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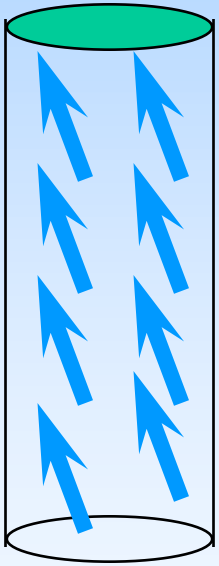
Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

Electrical dipoles on a thin long line



Available:

Thin line, infinitely long, homogeneously filled with dipoles, each with dipole moment ***p*** [Cm]

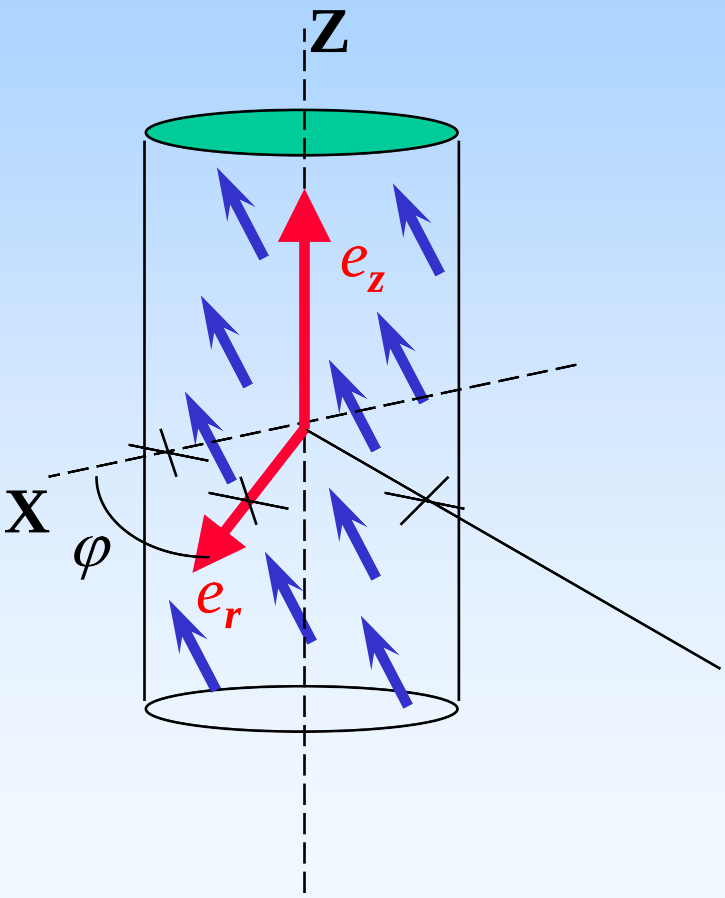
Question:

Calculate ***E***-field in arbitrary points around the line

Electrical dipoles on a thin long line

- Analysis and symmetry
- Approach to solution
- Calculations
- Conclusions

Analysis and Symmetry (1)



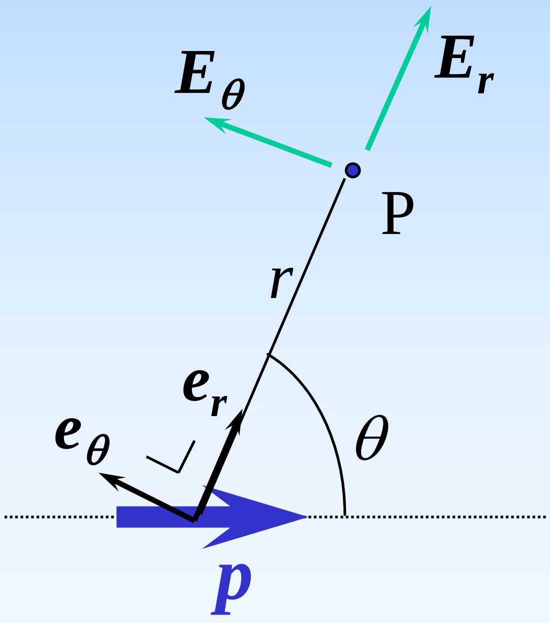
1. Cylinder: infinitely long and thin
2. Distribution of dipoles:
   - *n* dipoles / meter; homogeneous;
   - all directions uniform
   - each: dipole moment ***p*** [Cm]
3. Coordinate axes: X,Y,Z

   Z-axis = symm. axis
4. Cylinder symmetry:

   all points at equal *r* are equivalent, even if at different *z* or φ

Analysis and Symmetry (2)

Assume: field components of dipole field at some distance in point P àre known.



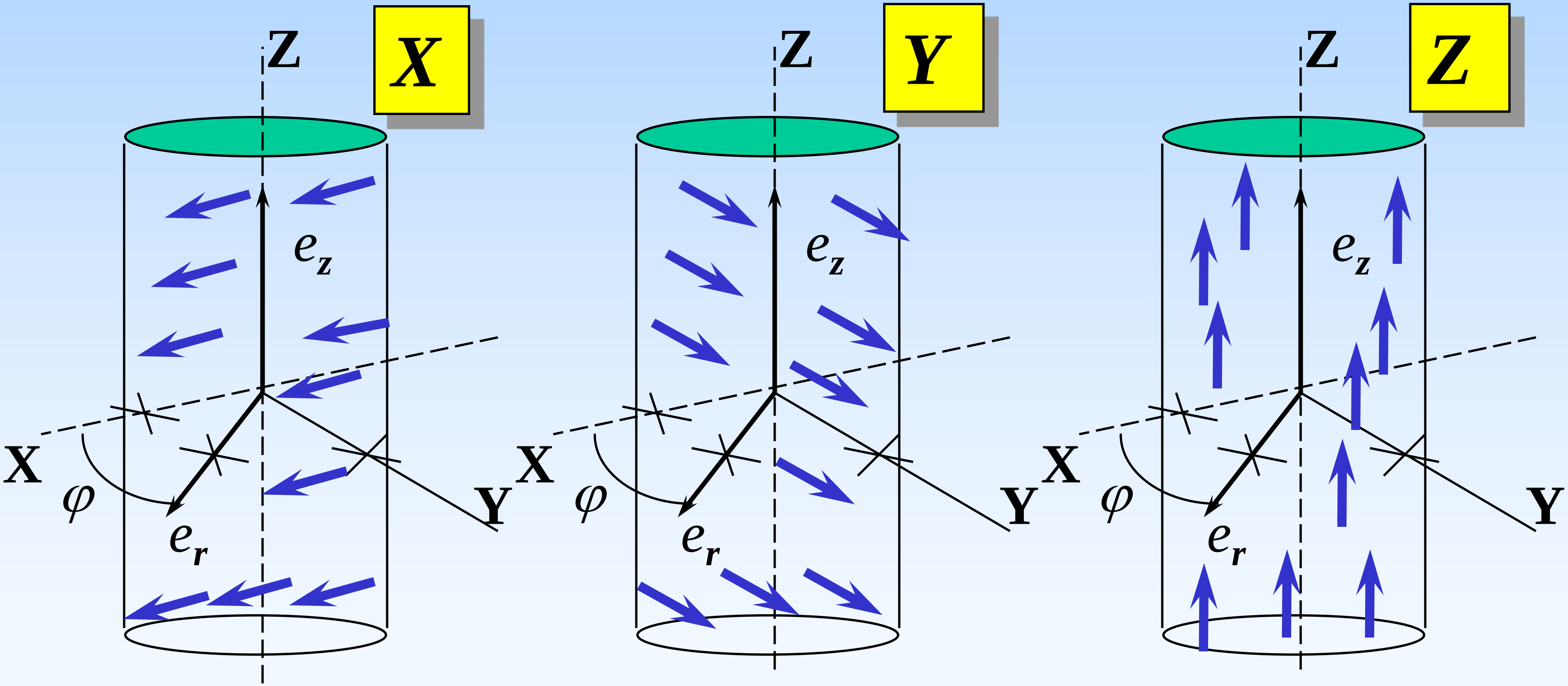
Dipole :

$$E_r = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$$

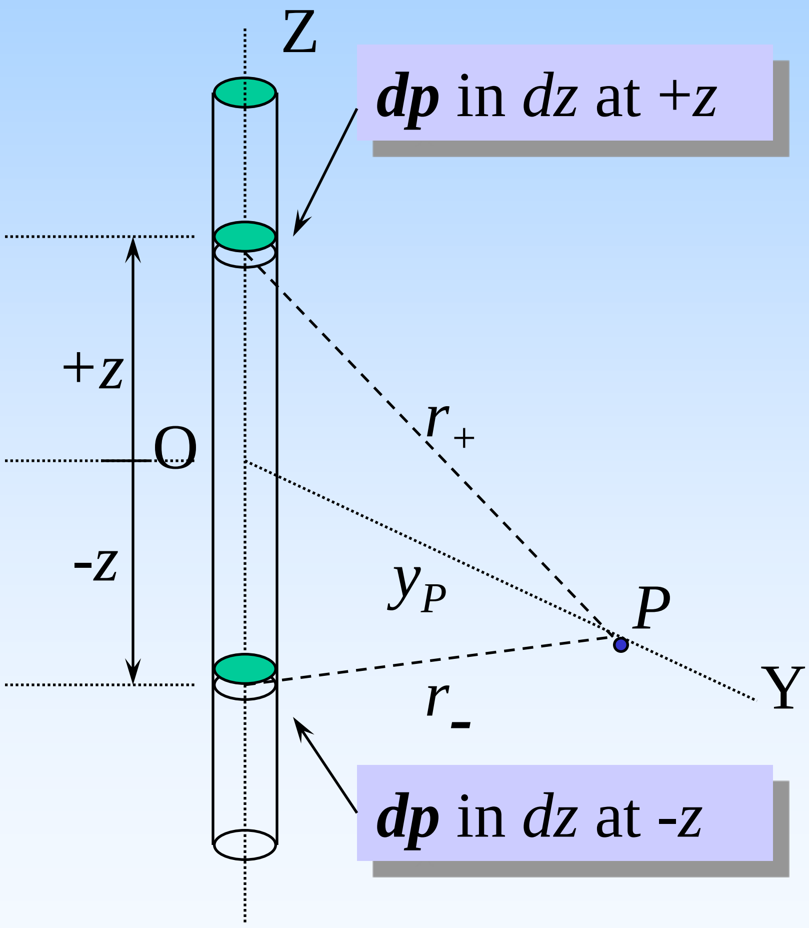
$$E_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$$

Analysis and Symmetry (3)

Possible directions of the dipoles:



Approach to solution



n **=** dipole density (per meter)

<u>Choose</u>: Point *P* at distance y_P from axis.

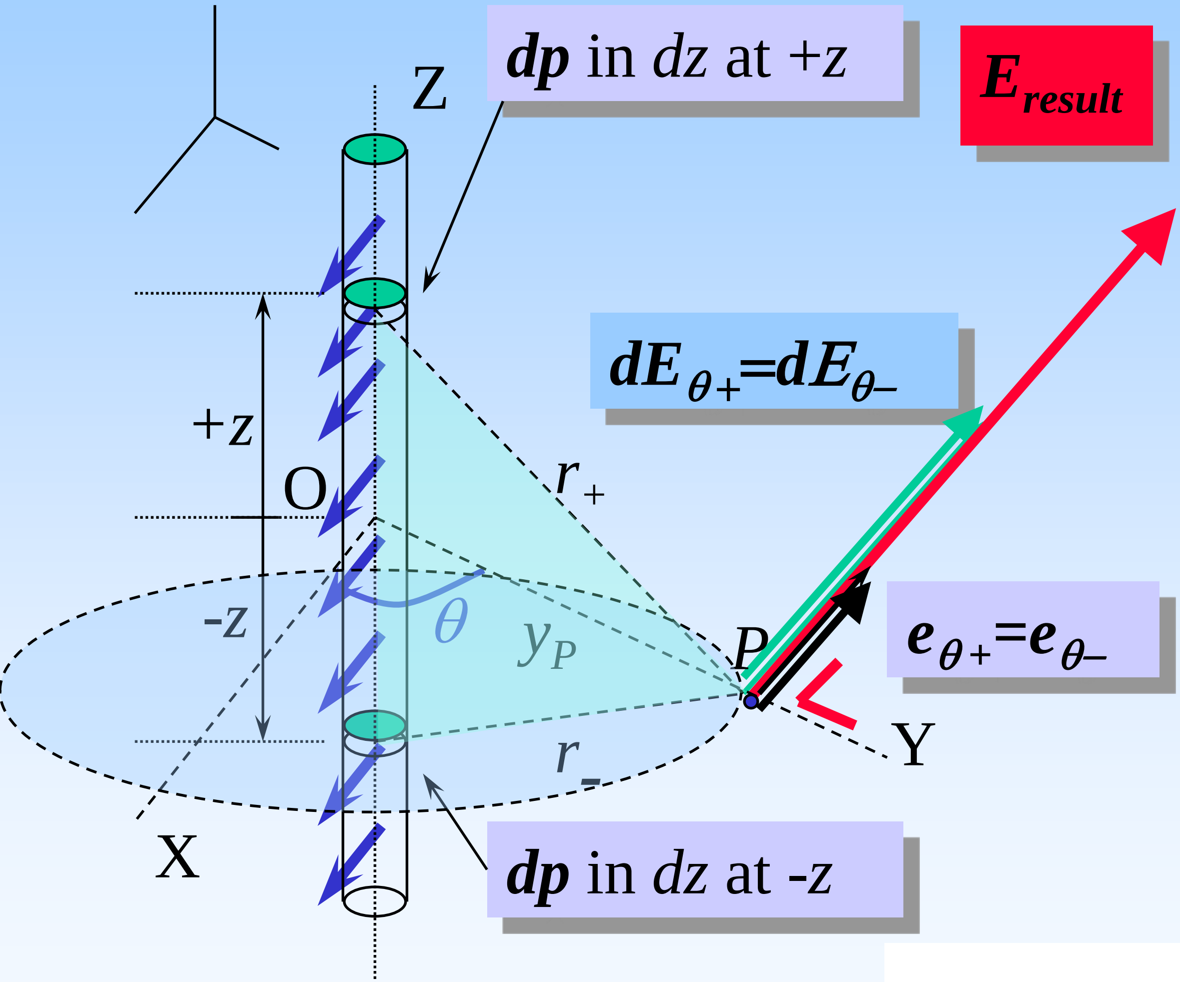
<u>Use symmetry:</u>

Dipole elements at +z and -z will contribute symmetrically

$\mathbf{dp} = n\,\mathbf{p}\,dz$

Dipoles // X-axis

$$E_r = \frac{2p.\cos\theta}{4\pi\varepsilon_0 r^3} \; ; \; E_\theta = \frac{p.\sin\theta}{4\pi\varepsilon_0 r^3}$$



dp = *n* ***p*** *dz*

Case 1:

all dipoles // +X-axis

$\boldsymbol{\theta = 90^0}$ for all dipoles

$$dE_r = \frac{2dp\cos\theta}{4\pi\varepsilon_0 r^3} = 0$$

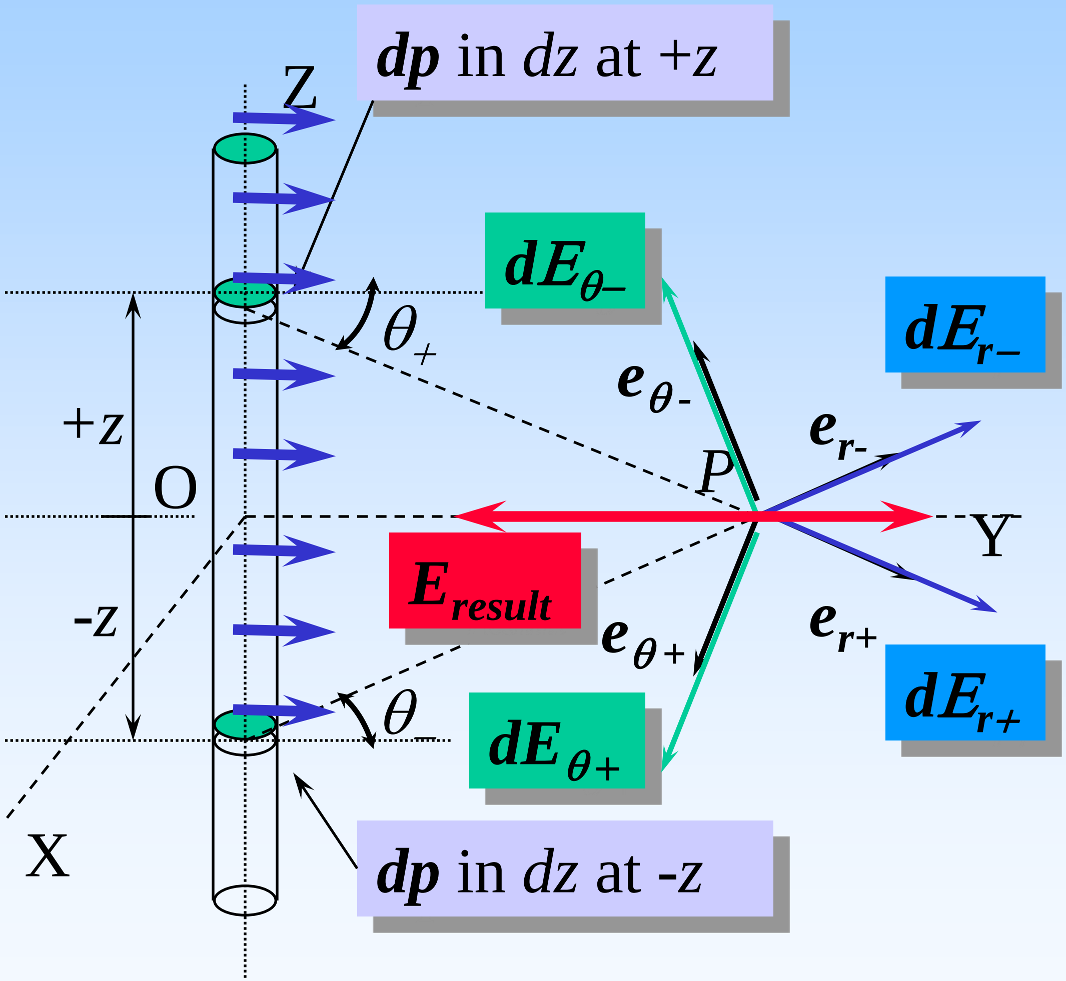
$$dE_\theta = \frac{dp\sin\theta}{4\pi\varepsilon_0 r^3} = \frac{dp}{4\pi\varepsilon_0 r^3}$$

==> ***E // -X-axis***

integratio n with $r^2 = {y_p}^2 + z^2 : E_\theta = \dfrac{np}{2\pi\varepsilon_0 {y_p}^2}$

Dipoles // Y-axis

$$E_r = \frac{2p.\cos\theta}{4\pi\varepsilon_0 r^3}\ ;\ E_\theta = \frac{p.\sin\theta}{4\pi\varepsilon_0 r^3}$$



Case 2:

all dipoles // +Y-axis

Find directions of the field contributions ***dE***

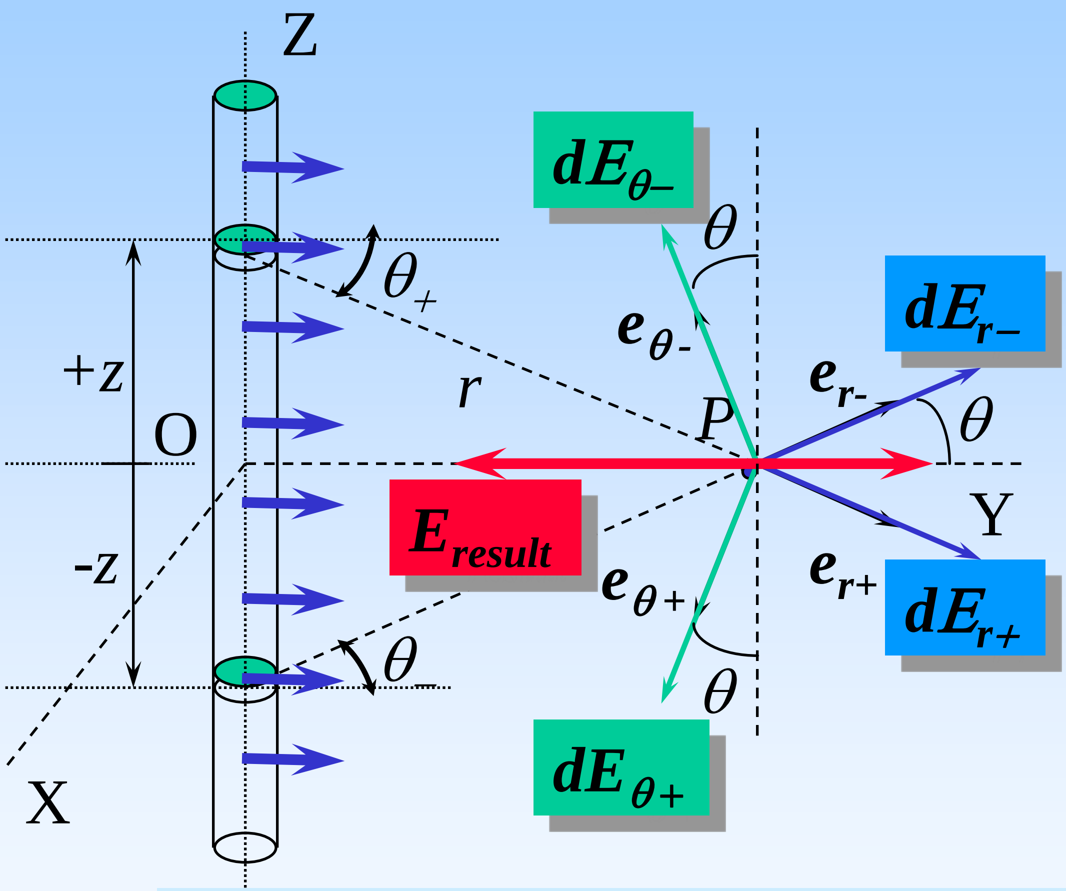
dipole	+z	-z
$dE_r \sim \cos\theta$	>0	>0
$dE_\theta \sim \sin\theta$	>0	>0

resulting vector :

==> *E // Y-axis*

Dipoles // Y-axis

$$E_r = \frac{2p.\cos\theta}{4\pi\varepsilon_0 r^3} \; ; \; E_\theta = \frac{p.\sin\theta}{4\pi\varepsilon_0 r^3}$$



$$dp = n.p.dz \quad ; \quad \theta_+ = \theta_- = \theta$$

Y-components:

dE_{r+} and dE_{r-} : each:

$$\left(\frac{np\,dz}{4\pi\varepsilon_0 r^3} 2\cos\theta \right) \cos\theta$$

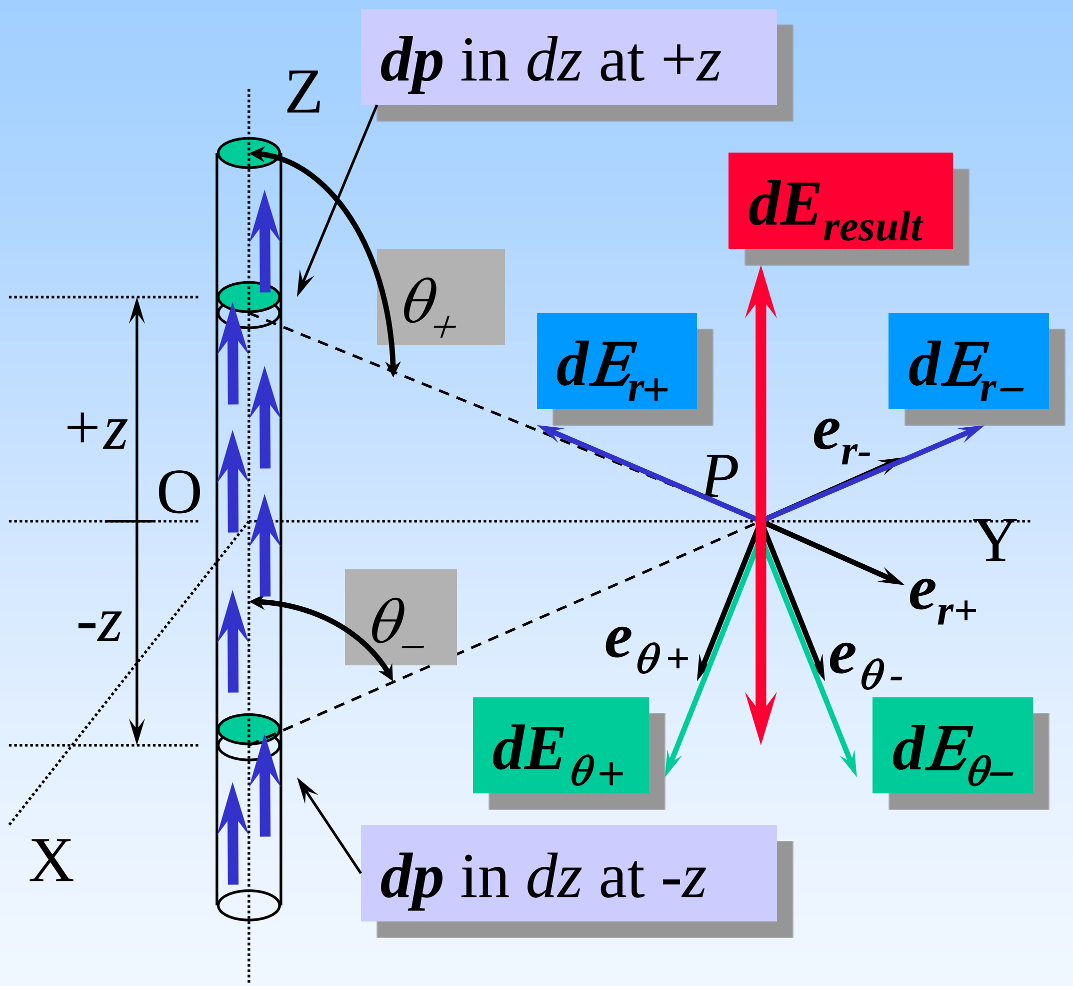
$dE_{\theta+}$ and $dE_{\theta-}$: each:

$$-\left(\frac{np\,dz}{4\pi\varepsilon_0 r^3} \sin\theta \right) \sin\theta$$

The integrations can be performed using standard integrals of rational functions. The resulting ***E***-vector will be directed along the Y-axis.

Dipoles // Z-axis

$$E_r = \frac{2p.\cos\theta}{4\pi\varepsilon_0 r^3}\ ;\ E_\theta = \frac{p.\sin\theta}{4\pi\varepsilon_0 r^3}$$



Case 3:

all dipoles // +Z-axis

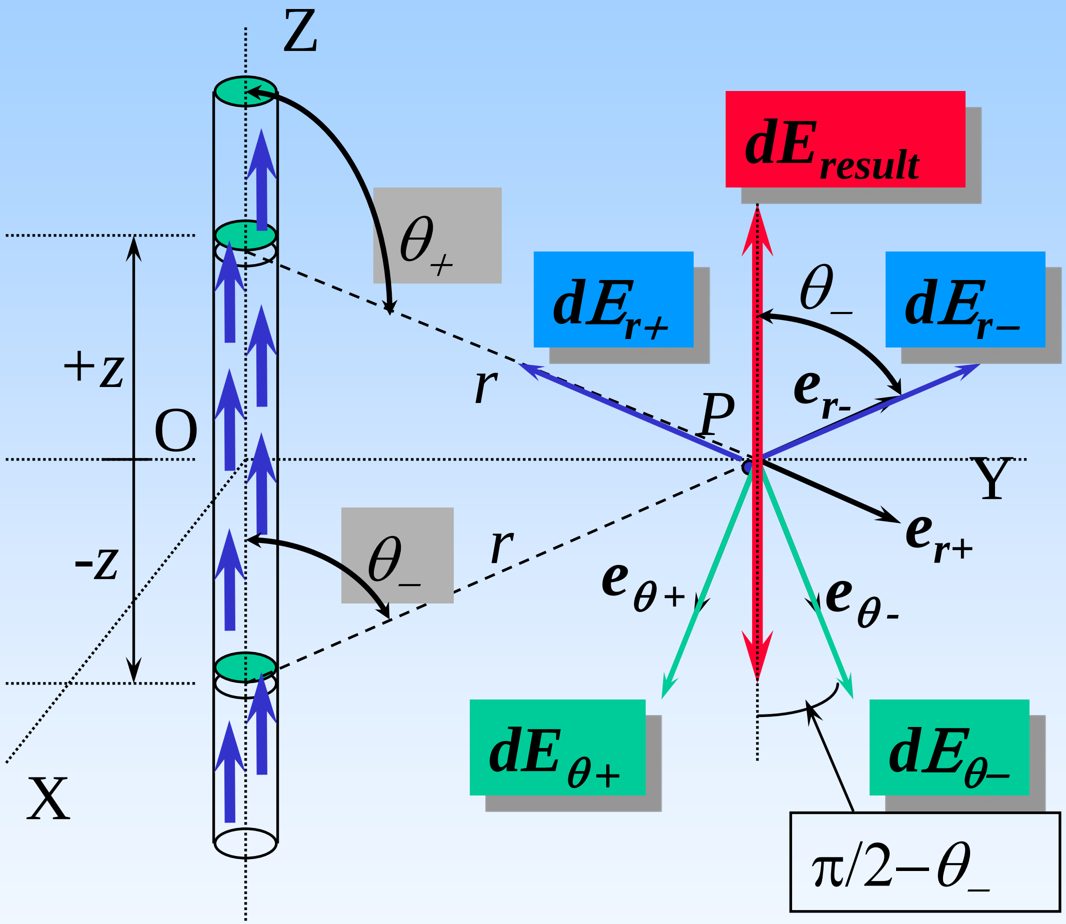
dipool	+	-
$dE_r \sim \cos\theta$	<0	>0
$dE_\theta \sim \sin\theta$	>0	>0

<u>resulting vector :</u>

==> ***E // Z-axis***

Dipoles // Z-axis

$$E_r = \frac{2p.\cos\theta}{4\pi\varepsilon_0 r^3}\ ;\ E_\theta = \frac{p.\sin\theta}{4\pi\varepsilon_0 r^3}$$



$$dp = n\,p\,dz$$

$$\theta_+ = \pi - \theta_-$$

Z-components:

dE_{r+} and dE_{r-} : each:

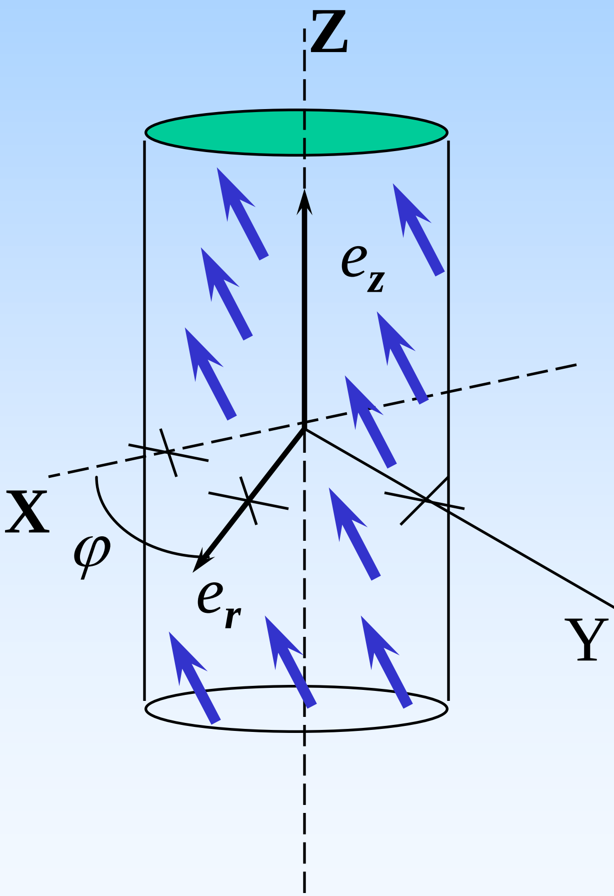
$$\left(\frac{np\,dz}{4\pi\varepsilon_0 r^3}2\cos\theta_-\right)\cos\theta_-$$

$dE_{\theta+}$ and $dE_{\theta-}$: each:

$$-\left(\frac{np\,dz}{4\pi\varepsilon_0 r^3}\sin\theta_-\right)\sin\theta_-$$

The integrations can be performed using standard integrals of rational functions.
The resulting ***E***-vector will be directed along the Z-axis.

Conclusions



Dipole direction	***dE*** – direction in point P on Y-axis
X	$\mathbf{dE} = -\dfrac{np\,dz}{2\pi\varepsilon_0 r^3}\mathbf{e_x}$
Y	$\mathbf{dE} = \dfrac{np\,dz}{4\pi\varepsilon_0 r^3}\left(2\cos^2\theta - \sin^2\theta\right)\mathbf{e_y}$
Z	$\mathbf{dE} = \dfrac{np\,dz}{4\pi\varepsilon_0 r^3}\left(2\cos^2\theta - \sin^2\theta\right)\mathbf{e_z}$

(θ and r are measured from dipole to P)

the end