Electric Field of a thin Disk

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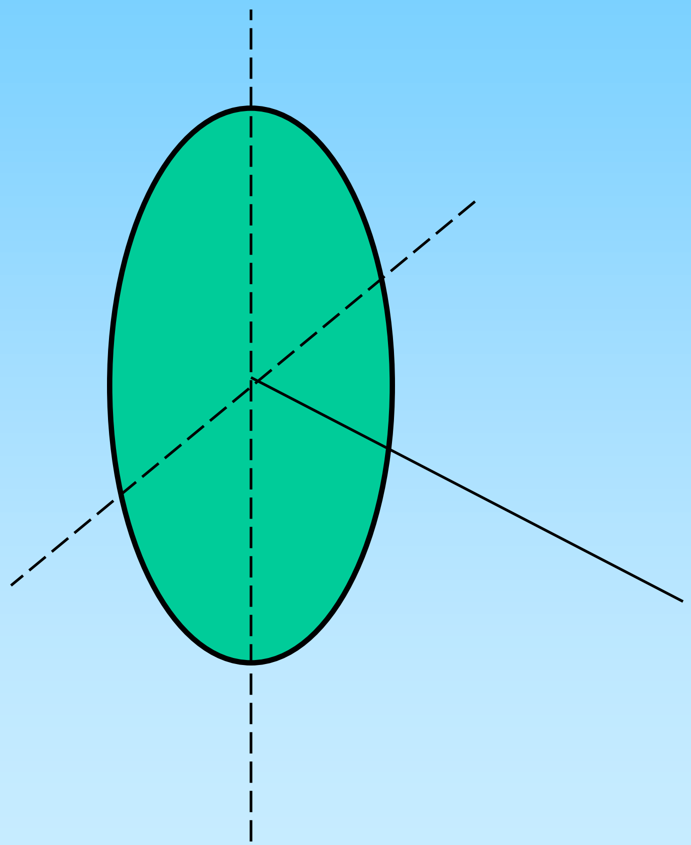
Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

E-field of a thin disk



<u>Available :</u>

A thin circular disk with radius *R* and charge density σ [C/m^2]

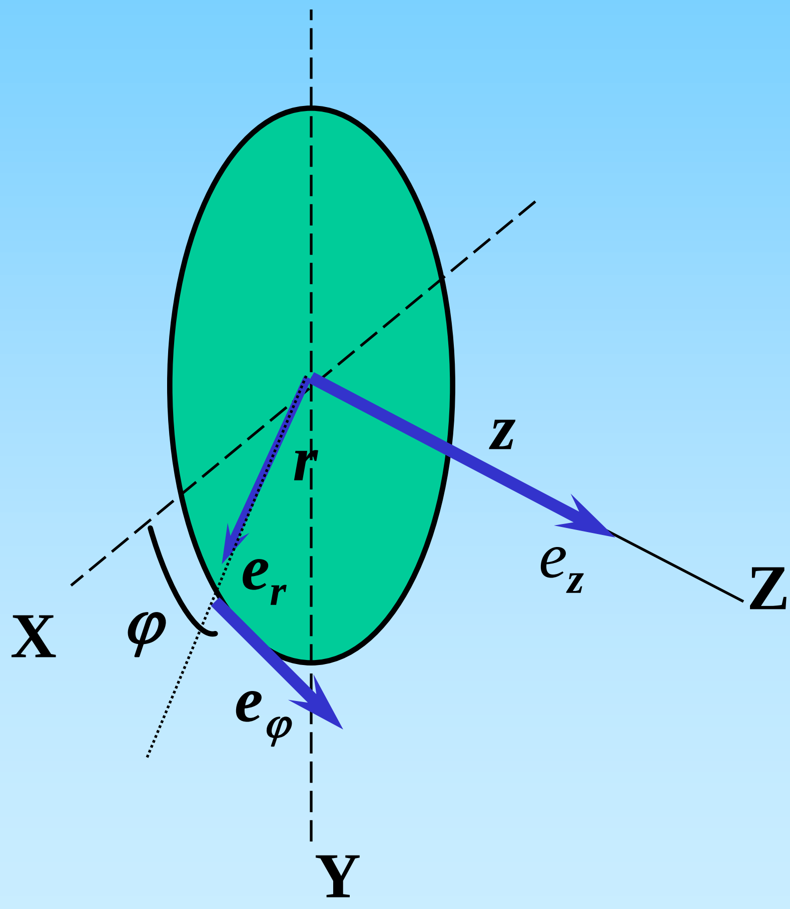
Question :

Calculate ***E***-field in arbitrary points a both sides of the disk

E-field of a thin disk

- Analysis and symmetry
- Approach to solution
- Calculations
- Conclusions
- Appendix: angular integration

Analysis and Symmetry



1. Charge distribution:

   σ [C/m^2] homogeneous

2. Coordinate axes:

   Z-axis = symmetry axis,

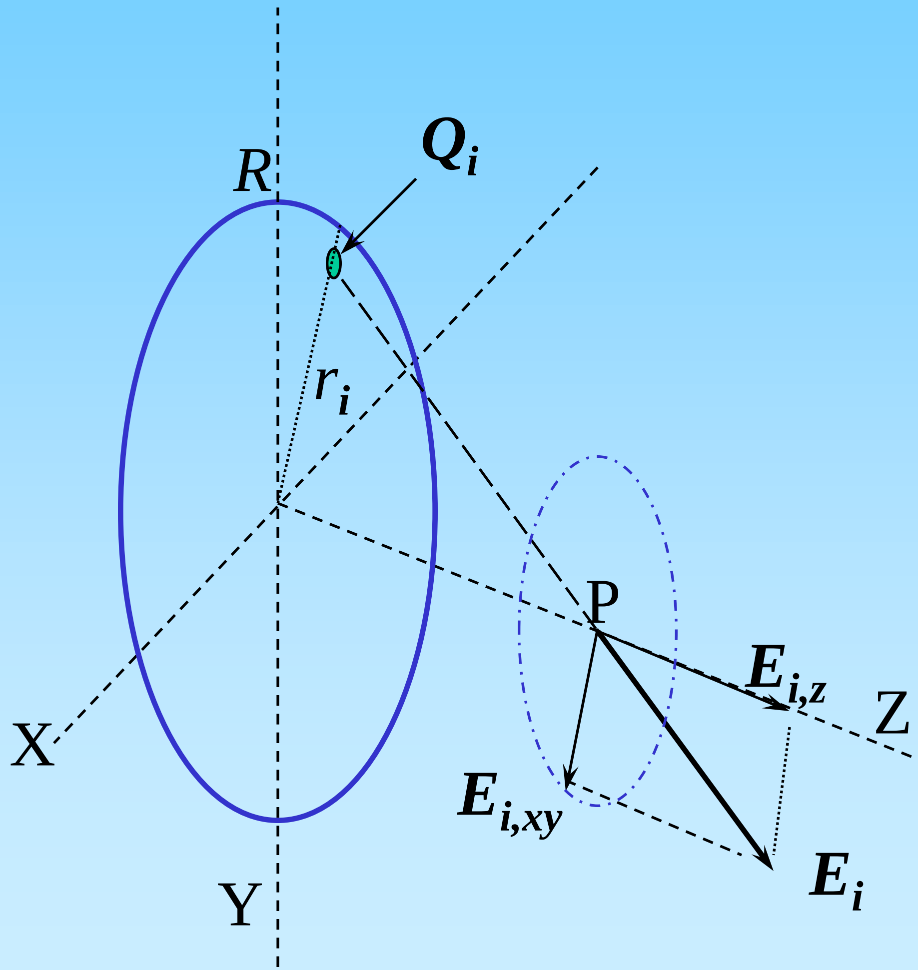
   perpendicular to disk

3. Symmetry: cylinder

4. Cylinder coordinates:

   r*, *z*, *φ

Analysis, field build-up



1. XYZ-axes

2. Point P on Z-axis

3. all Q_i's at r_i *and* φ_i contribute $\boldsymbol{E_i}$ to $\boldsymbol{E}$ in P

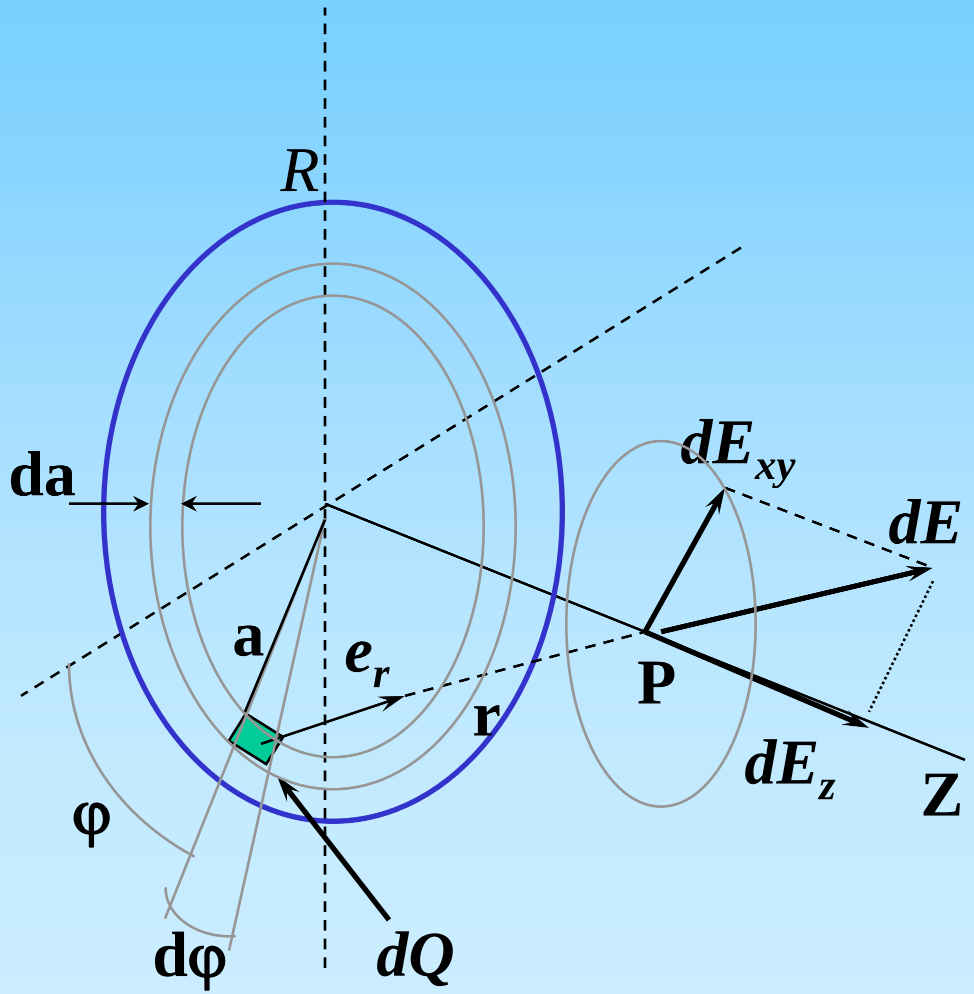
4. $\boldsymbol{E_{i,xy}}$, $\boldsymbol{E_{i,z}}$

5. expect: $\Sigma\, \boldsymbol{E_{i,xy}} = 0$,

to be checked !!

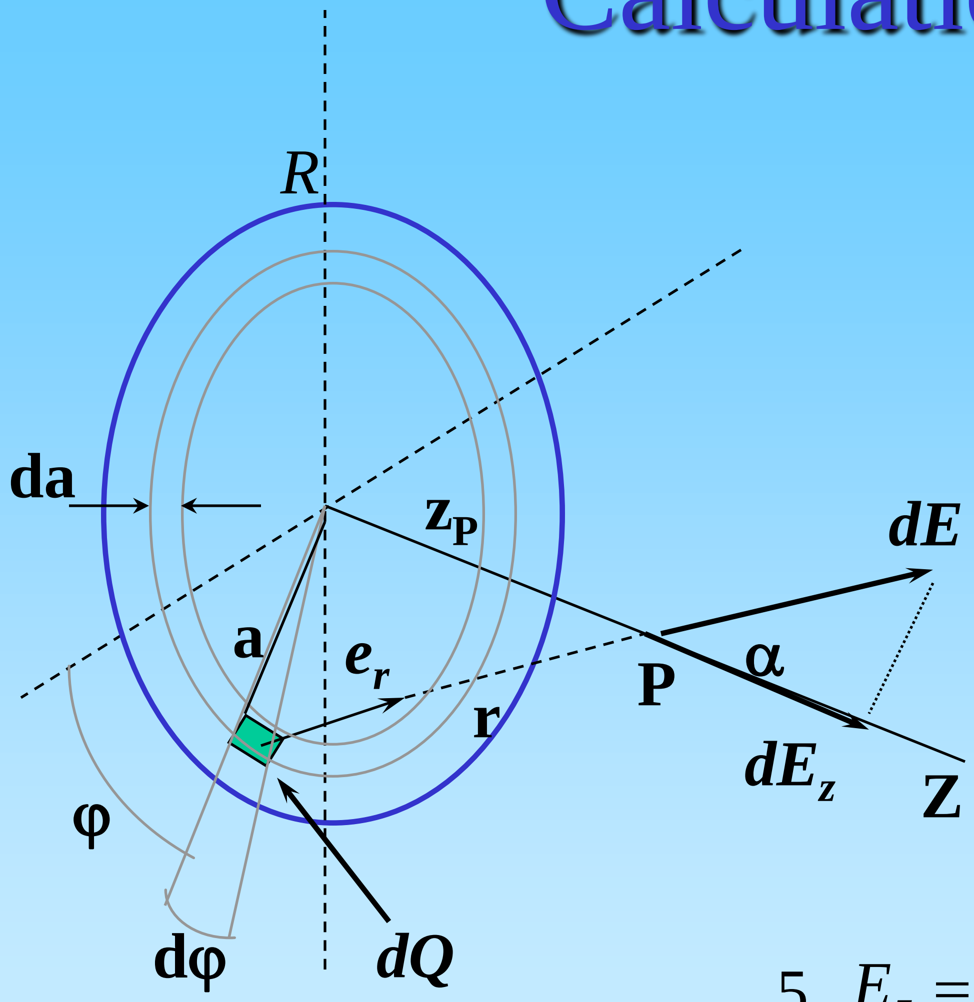
6. $\boldsymbol{E} = E_z\, \boldsymbol{e_z}$ ***only !***

Approach to solution



1. Rings and segments
2. Distributed charges
3. $$\boldsymbol{dE} = \frac{dQ}{4\pi\varepsilon_0 r^2}\boldsymbol{e_r}$$
4. $dQ = \sigma.dA = \sigma\,(da.)(a\,d\varphi)$
5. z- component only !
6. $$\boldsymbol{dE}_z = \frac{dQ}{4\pi\varepsilon_0 r^2}(\boldsymbol{e_r} \bullet \boldsymbol{e}_z)$$

Calculations (1)



1. $$\boldsymbol{dE}_z = \frac{dQ}{4\pi\varepsilon_0 r^2}(\boldsymbol{e_r} \bullet \boldsymbol{e}_z)$$

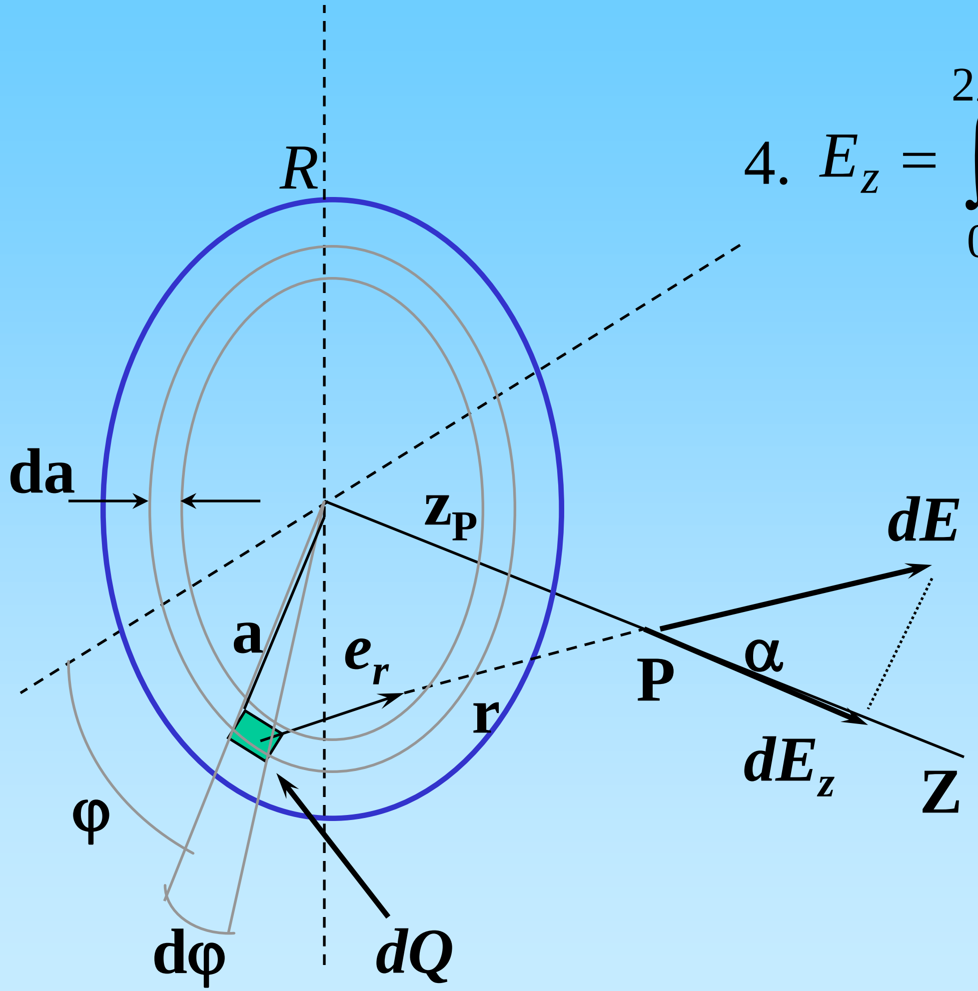
2. $$dQ = \sigma.dA = \sigma\,(da.)(a\,d\varphi)$$

3. $$\mathbf{e_r} \bullet \mathbf{e_z} = \cos\alpha = \frac{z_P}{\sqrt{a^2 + z_P{}^2}}$$

$$r^2 = a^2 + z_P{}^2$$

5. $$E_z = \int_0^{2\pi}\int_0^{R} \frac{\sigma.da.a.d\varphi}{4\pi\varepsilon_0\left(a^2 + z_P{}^2\right)} \frac{z_P}{\sqrt{a^2 + z_P{}^2}}$$

Calculations (2)



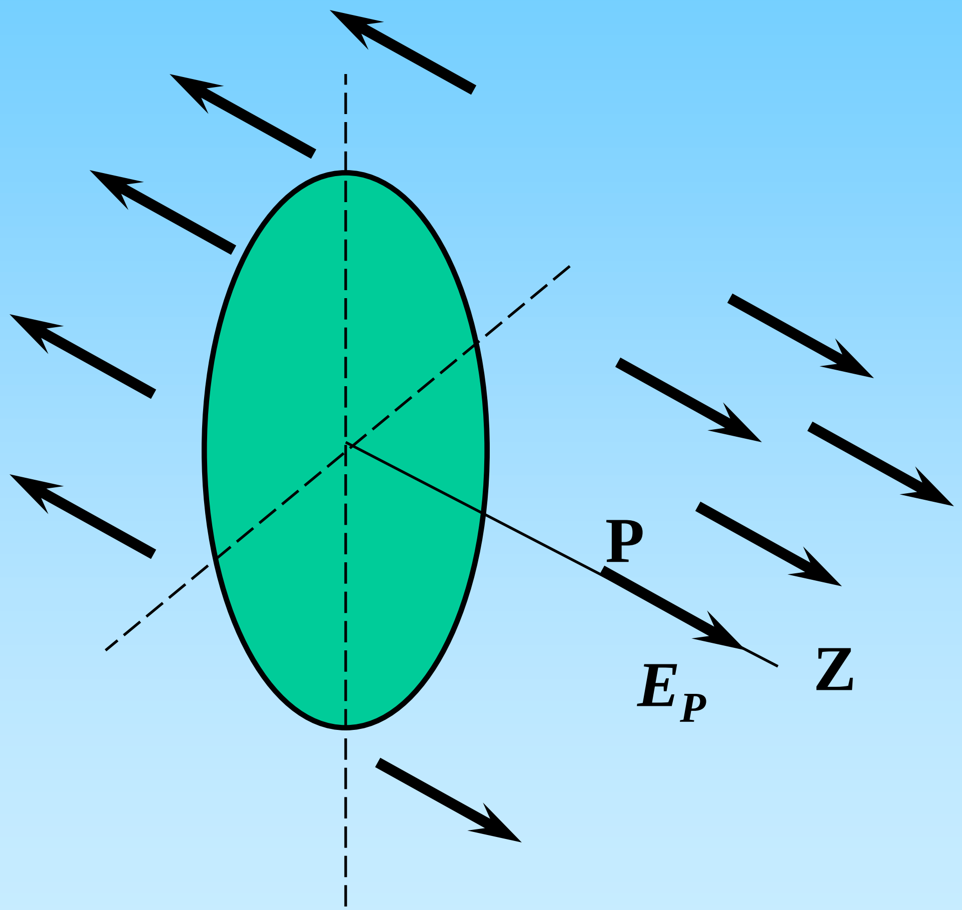
4. $$E_z = \int_0^{2\pi}\int_0^R \frac{\sigma.da.a.d\varphi}{4\pi\varepsilon_0\left(a^2+z_P{}^2\right)}\frac{z_P}{\sqrt{a^2+z_P{}^2}}$$

5. $$E_z = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{y_P}{\sqrt{y_P{}^2+R^2}}\right]$$

6. If R –> *infinity* :

$$E_z = \frac{\sigma}{2\varepsilon_0}$$

Conclusions



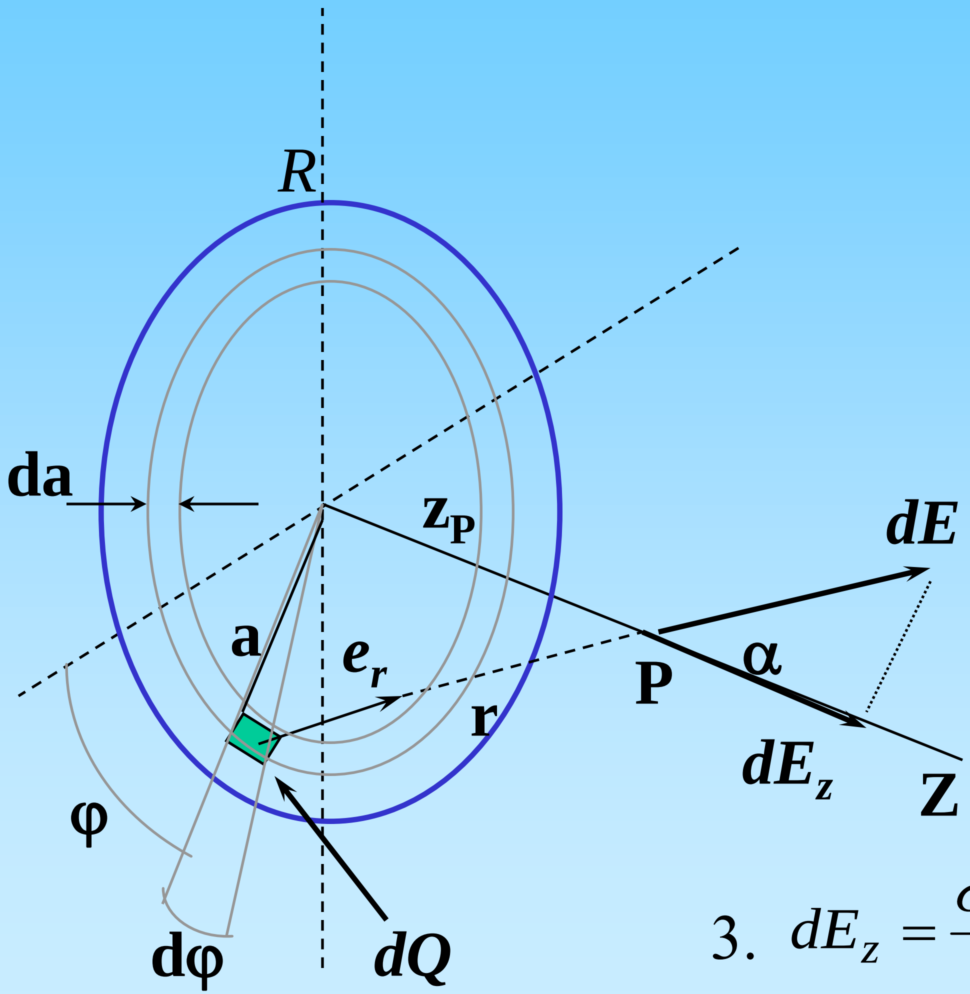
for infinite disk:

$$\boldsymbol{E}_P = \frac{\sigma}{2\varepsilon_0}\boldsymbol{e_z}$$

field strength independent of distance to disk =>

homogeneous field

Appendix: angular integration (1)



1. $$\mathbf{dE}_z = \frac{dQ}{4\pi\varepsilon_0 r^2}(\mathbf{e_r} \bullet \mathbf{e}_z)$$

2. $$dQ = \sigma.dA = \sigma\,(da.)(a\,d\varphi)$$

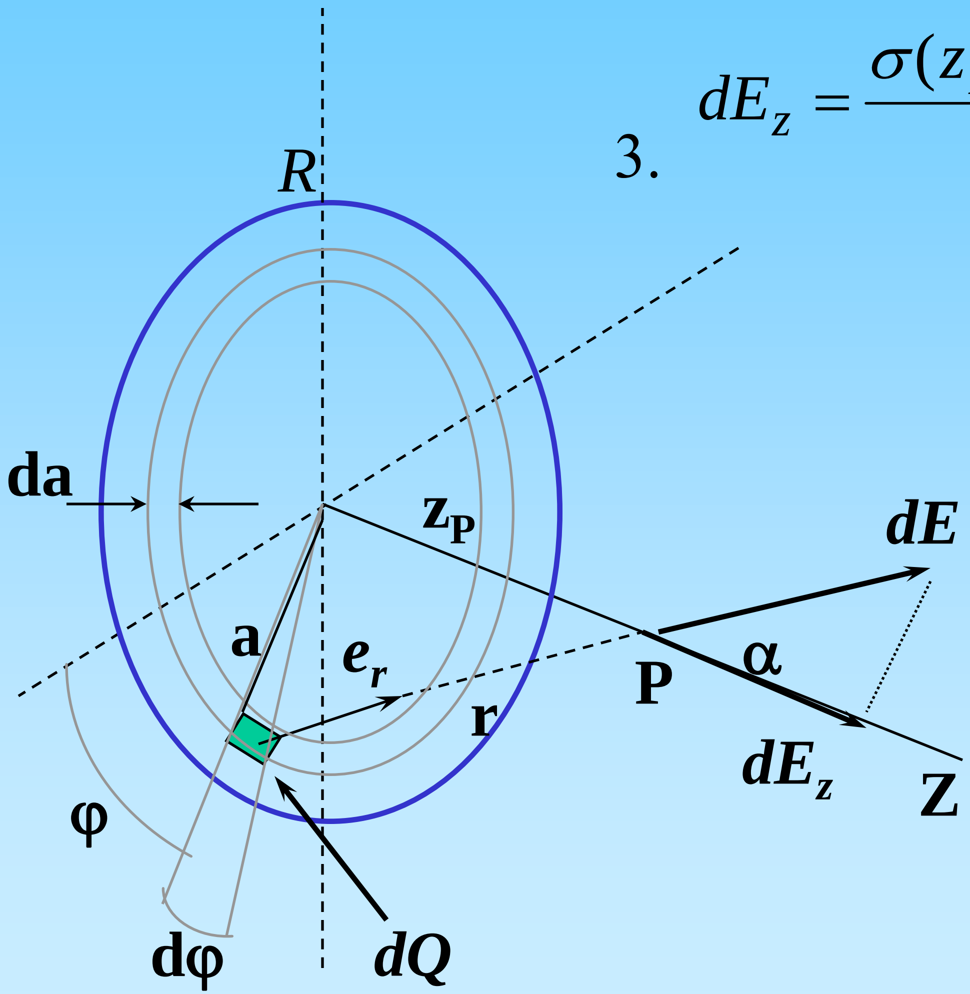
$$\mathbf{e_r} \bullet \mathbf{e_z} = \cos\alpha$$

$$r = f(\alpha) = \frac{z_P}{\cos\alpha}$$

$$a = z_P \tan\alpha \qquad da = \frac{z_P}{\cos^2\alpha}d\alpha$$

3. $$dE_z = \frac{\sigma(z_P\tan\alpha)(z_P\cos^{-2}\alpha d\alpha)d\varphi\cos\alpha}{4\pi\varepsilon_0(z_P{}^2\cos^{-2}\alpha)}$$

Appendix: angular integration (2)



3. $$dE_z = \frac{\sigma(z_P \tan\alpha)(z_P \cos^{-2}\alpha d\alpha)d\varphi\cos\alpha}{4\pi\varepsilon_0(z_P{}^2\cos^{-2}\alpha)}$$

4. $$E_z = \int_0^R \frac{2\pi\sigma\sin\alpha d\alpha}{4\pi\varepsilon_0}$$

$$.. = \frac{\sigma}{2\varepsilon_0}(-1)\left[\cos\alpha_{max} - \cos 0\right]$$

$$.. = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z_P}{\sqrt{z_P{}^2 + R^2}}\right]$$

$$.. \xrightarrow{R\to\infty} \frac{\sigma}{2\varepsilon_0}$$

the end