The Electric Field of a Dipole

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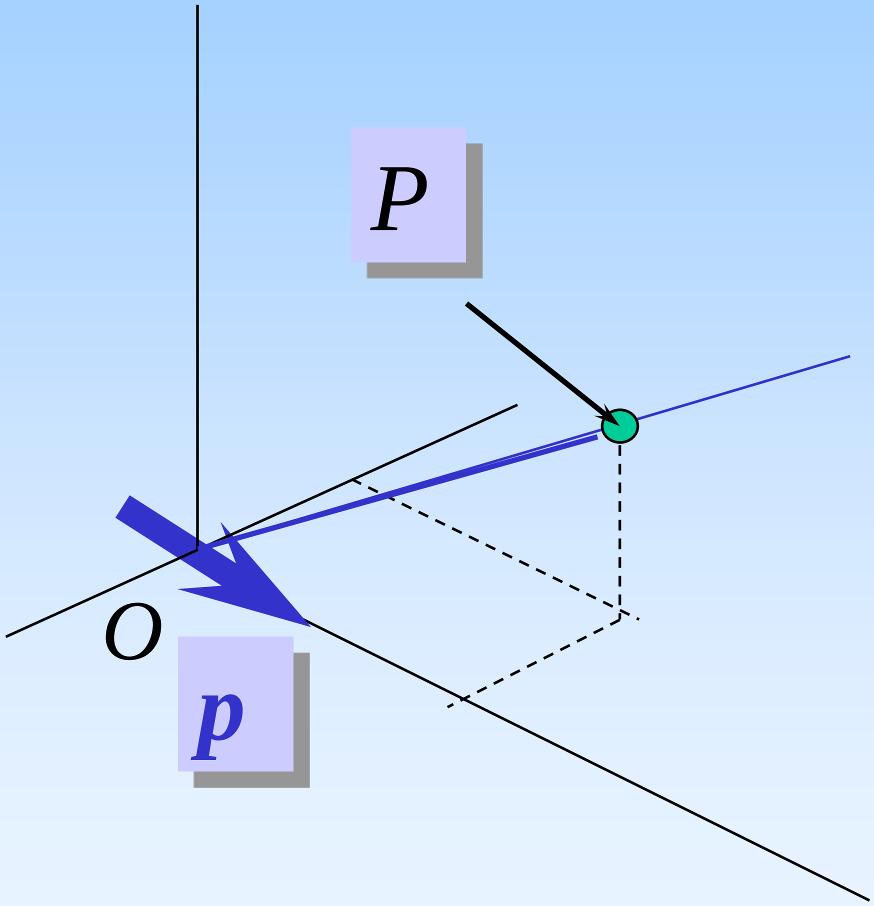
Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

The Electric Field of a Dipole



<u>Given:</u>

Electric Dipole, situated in O, with dipole moment ***p*** [Cm]

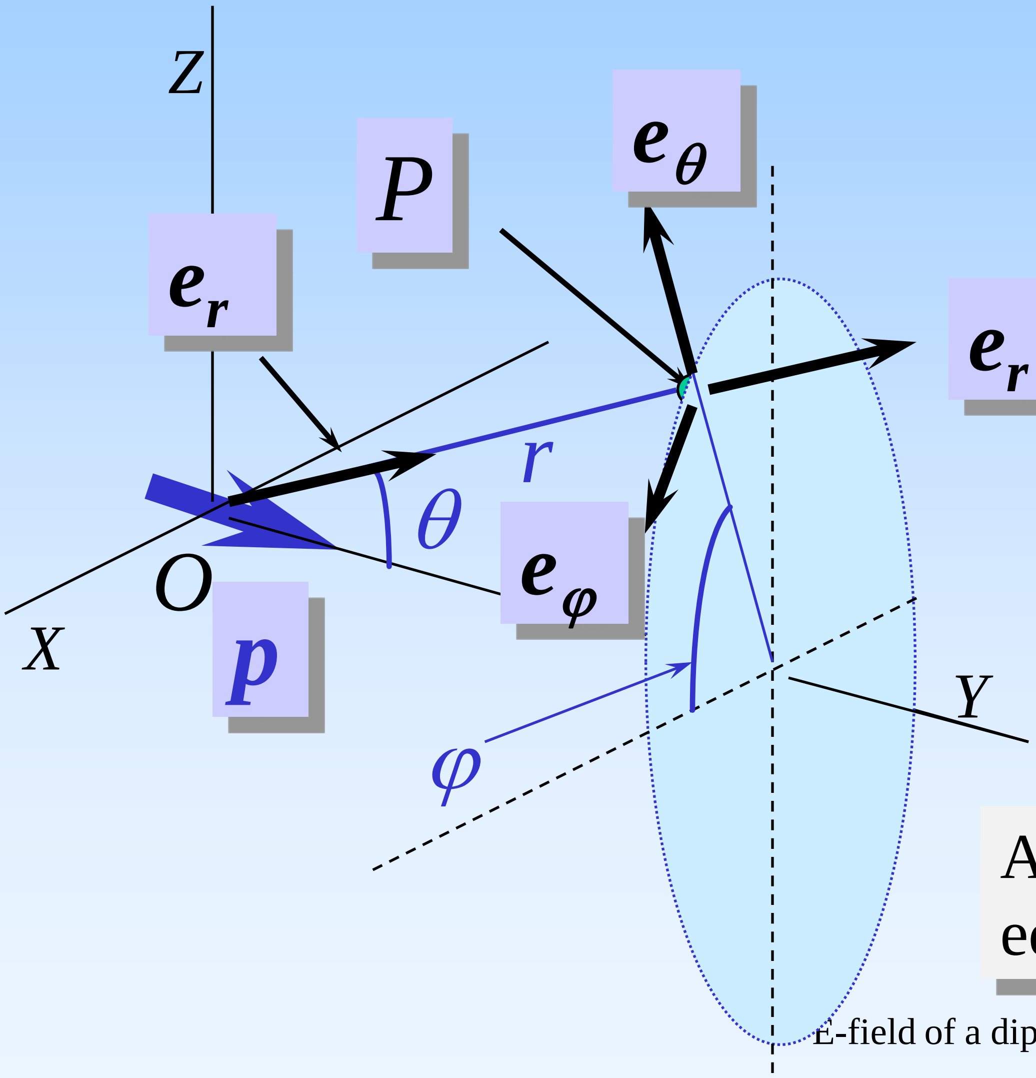
<u>Question:</u>

Calculate ***E***-field in arbitrary points P around the dipole

The Electric Field of a Dipole

- Analysis and symmetry
- Approach to solution
- Calculations
- Conclusions
- Field lines in XY-plane

Analysis and Symmetry



<u>Coordinate axes:</u> X, Y, Z

Y-axis = dipolar axis

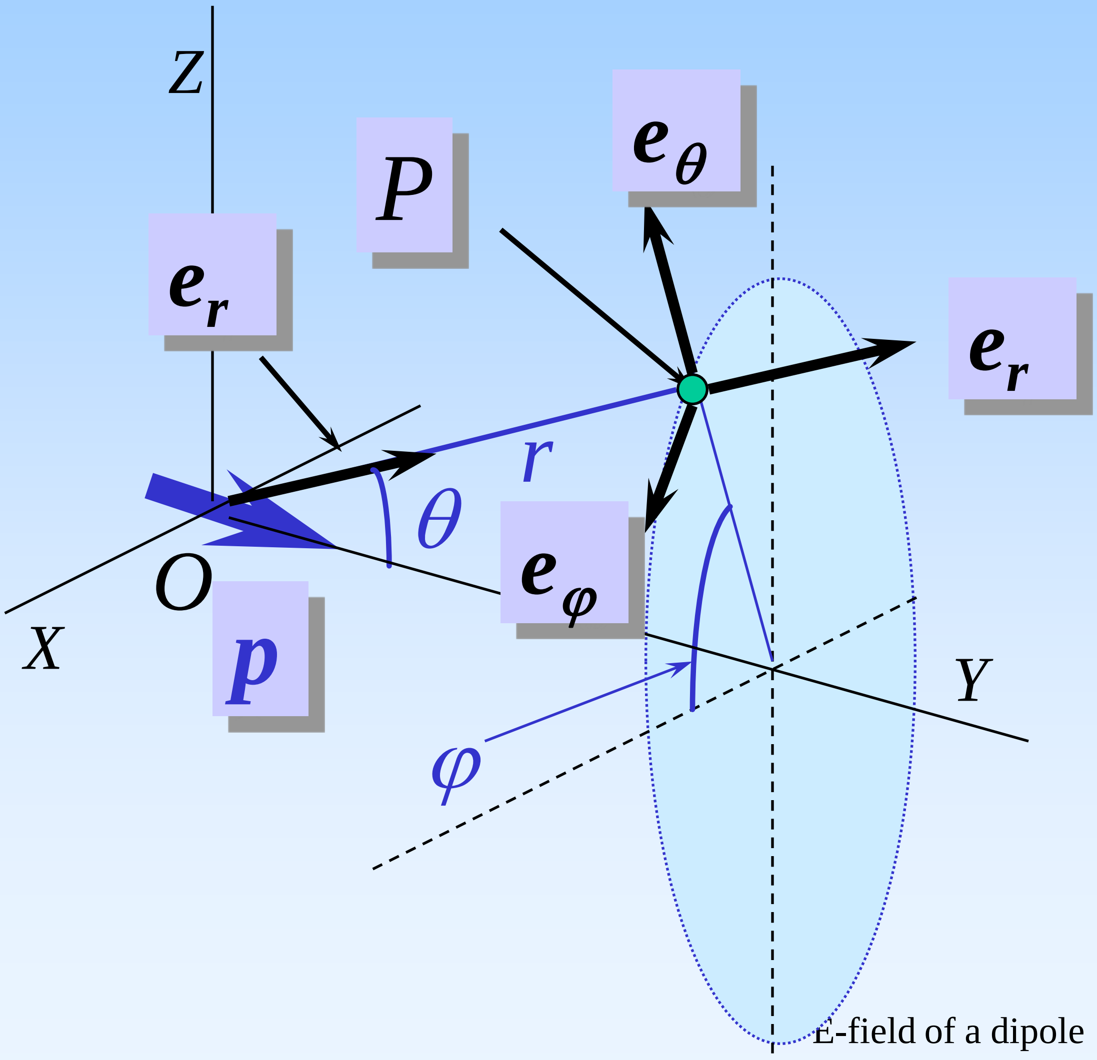
<u>Symmetry:</u> cylindrical around Y-axis

Cylindrical coordinates:

r, θ and φ (perpendicular')

All points at <u>equal r and θ</u> are equivalent, even if at <u>different</u> φ

Approach to solution



Several approaches possible:

1. $\boldsymbol{E}$ with "Coulomb";

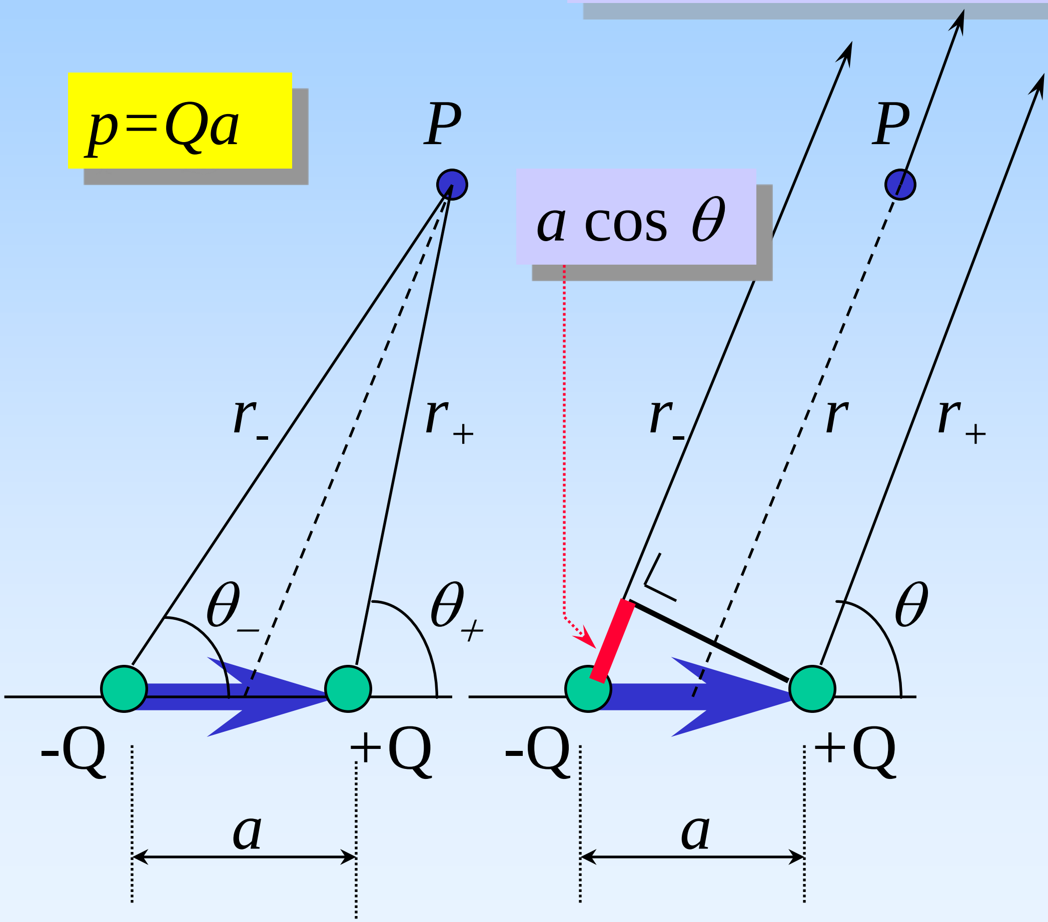
2. $\boldsymbol{E}$ from V with:
$\boldsymbol{E}$ = - grad V

Choose option 2.

Necessary: calculate $V(r,\theta,\varphi)$ first !

Intermezzo: $V(r,\theta,\varphi)$ -calculation

Calculate: Potential in P : (reference in inf.)



Contributions to V from $+Q$ and $-Q$:

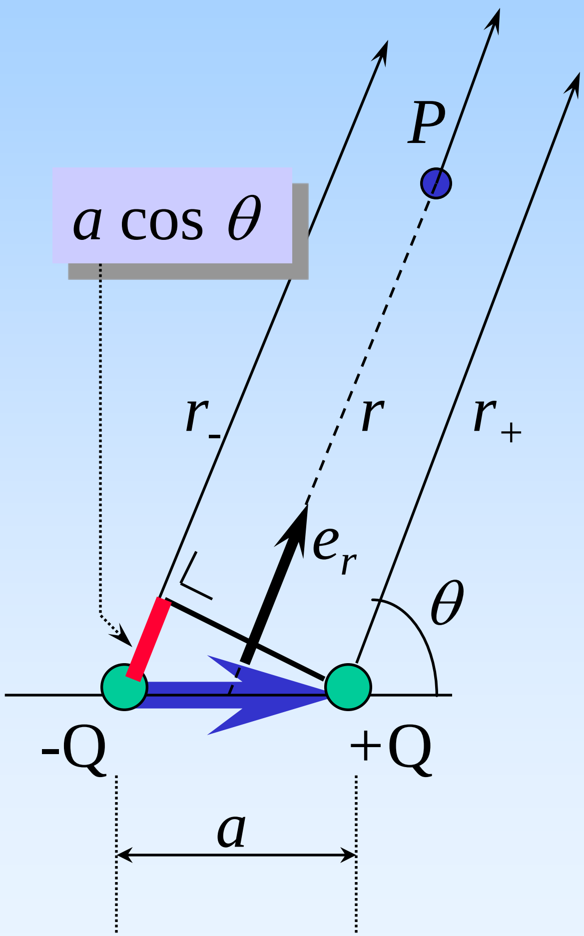
$$V(P) = \frac{+Q}{4\pi\varepsilon_0 r_+} + \frac{-Q}{4\pi\varepsilon_0 r_-}$$

$$= \frac{Q}{4\pi\varepsilon_0} \frac{r_+ - r_-}{r_+ r_-}$$

If $a << r_+, r_-$, then

$$V(r,\theta) = \frac{Q}{4\pi\varepsilon_0} \frac{a\cos\theta}{r^2}$$

Intermezzo: $V(r,\theta,\varphi)$ -calculation



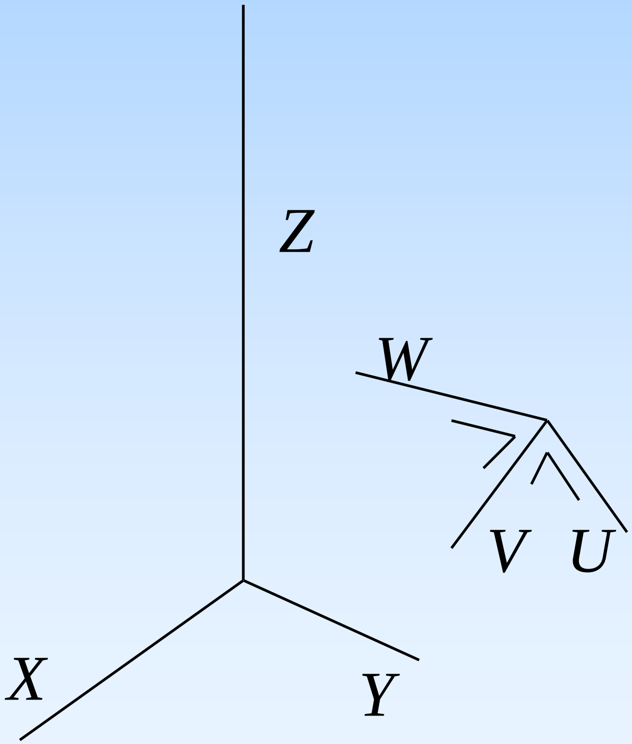
$$V(r,\theta) = \frac{Q}{4\pi\varepsilon_0}\frac{a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$V(r,\theta) = \frac{\boldsymbol{p} \bullet \boldsymbol{e_r}}{4\pi\varepsilon_0 r^2}$$

This approximation is called:

Far-field approximation

Calculation of *E* (1)



Calculate ***E*** from *V* using :

E **= -** grad *V*

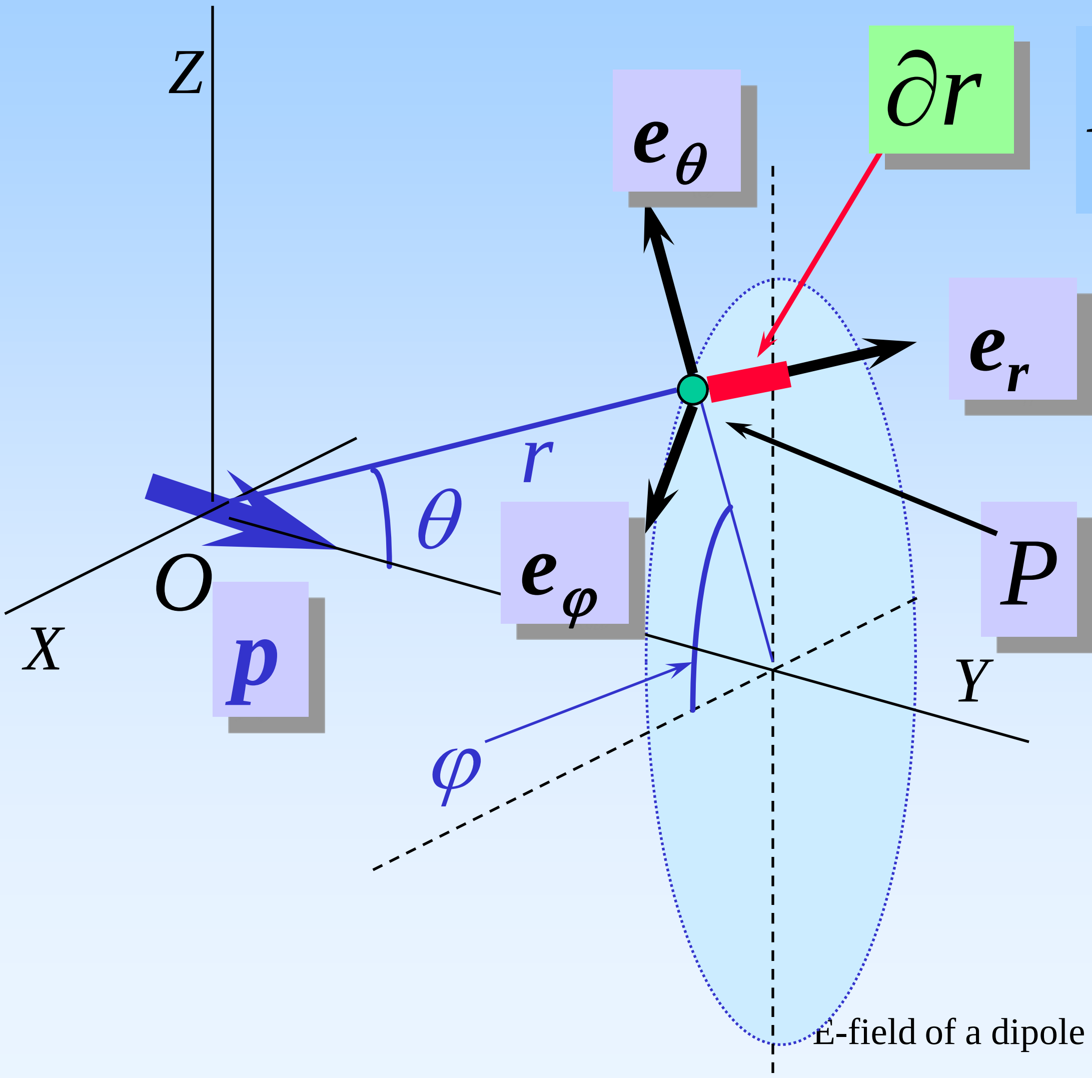
$$\boldsymbol{E} = -\frac{\partial V}{\partial x}\boldsymbol{e}_{\boldsymbol{x}} - \frac{\partial V}{\partial y}\boldsymbol{e}_{y} - \frac{\partial V}{\partial z}\boldsymbol{e}_{z}$$

In general :

$$\boldsymbol{E} = -\frac{\partial V}{\partial s_u}\boldsymbol{e}_{\boldsymbol{u}} - \frac{\partial V}{\partial s_v}\boldsymbol{e}_{v} - \frac{\partial V}{\partial s_w}\boldsymbol{e}_{w}$$

with s_u ,s_v and s_w line elements

Calculation of $\boldsymbol{E}$ (2)



E-field of a dipole

$$\boldsymbol{E} = -\frac{\partial V}{\partial s_u}\boldsymbol{e_u} - \frac{\partial V}{\partial s_v}\boldsymbol{e}_v - \frac{\partial V}{\partial s_w}\boldsymbol{e}_w$$

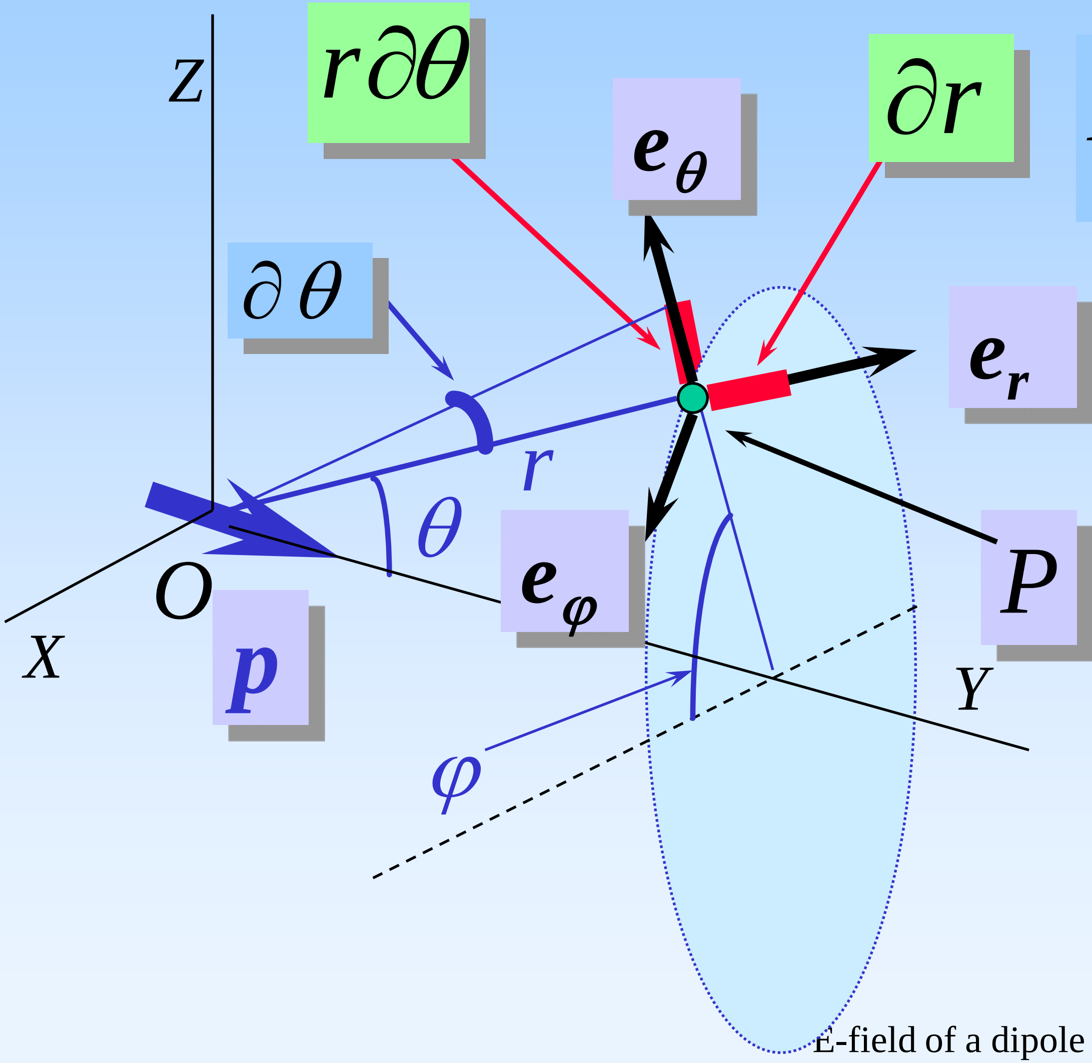
here $u=r,\ v=\theta\ ,w=\varphi$

$$\partial s_u = \partial r$$

$$\text{with}\quad V(r,\theta) = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\boldsymbol{E_r} = -\frac{\partial V}{\partial r}\boldsymbol{e_r} = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\boldsymbol{e_r}$$

Calculation of $\boldsymbol{E}$ (3)



E-field of a dipole

$$\boldsymbol{E} = -\frac{\partial V}{\partial s_u}\boldsymbol{e}_u - \frac{\partial V}{\partial s_v}\boldsymbol{e}_v - \frac{\partial V}{\partial s_w}\boldsymbol{e}_w$$

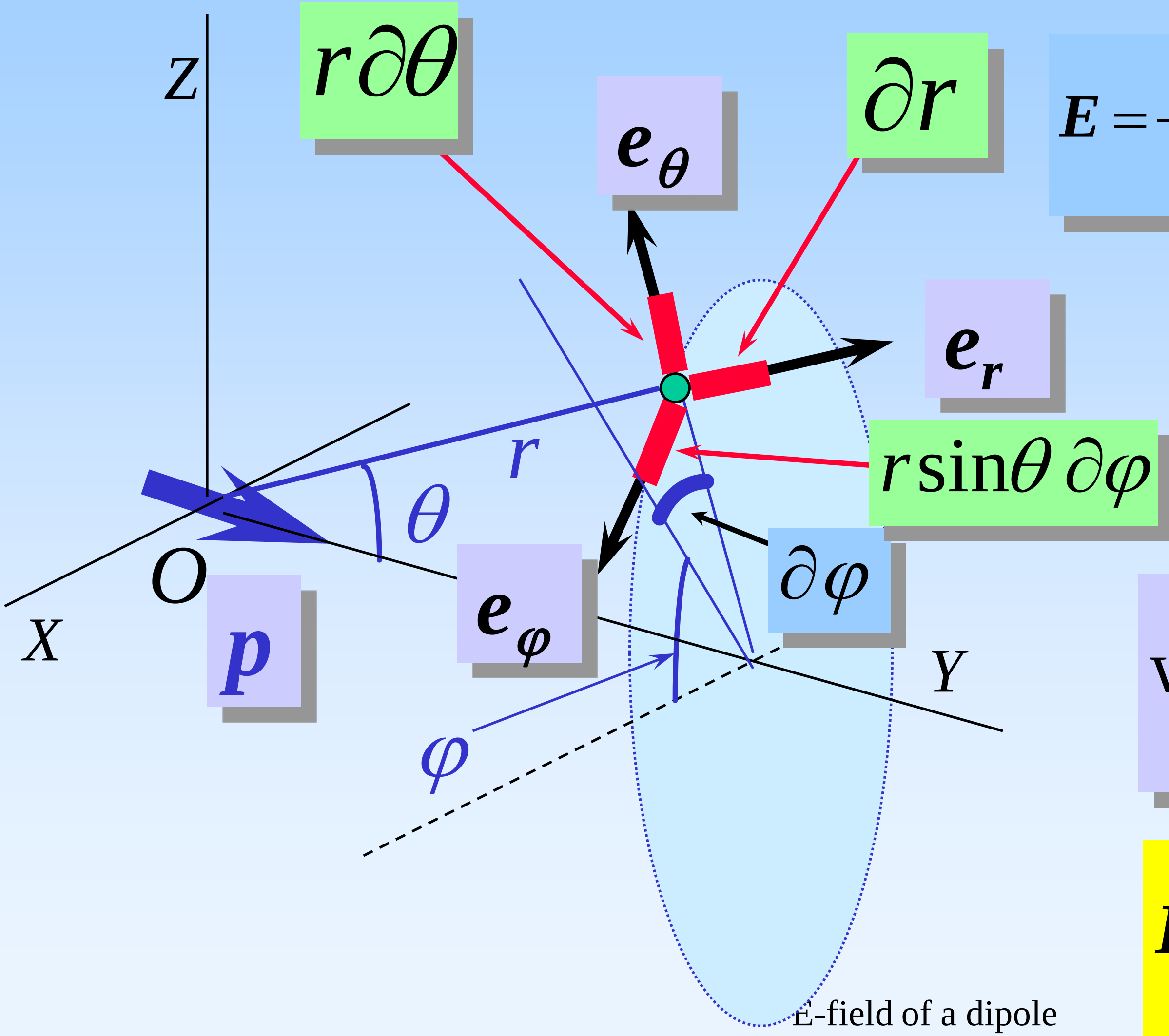
here $u=r$, $v=\theta$,$w=\varphi$

$$\partial s_v = r\,\partial\theta$$

with $\quad V(r,\theta) = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

$$\boldsymbol{E}_\theta = -\frac{\partial V}{r\partial\theta}\boldsymbol{e}_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\boldsymbol{e}_\theta$$

Calculation of $\boldsymbol{E}$ (4)



$$\boldsymbol{E} = -\frac{\partial V}{\partial s_u}\boldsymbol{e_u} - \frac{\partial V}{\partial s_v}\boldsymbol{e}_v - \frac{\partial V}{\partial s_w}\boldsymbol{e}_w$$

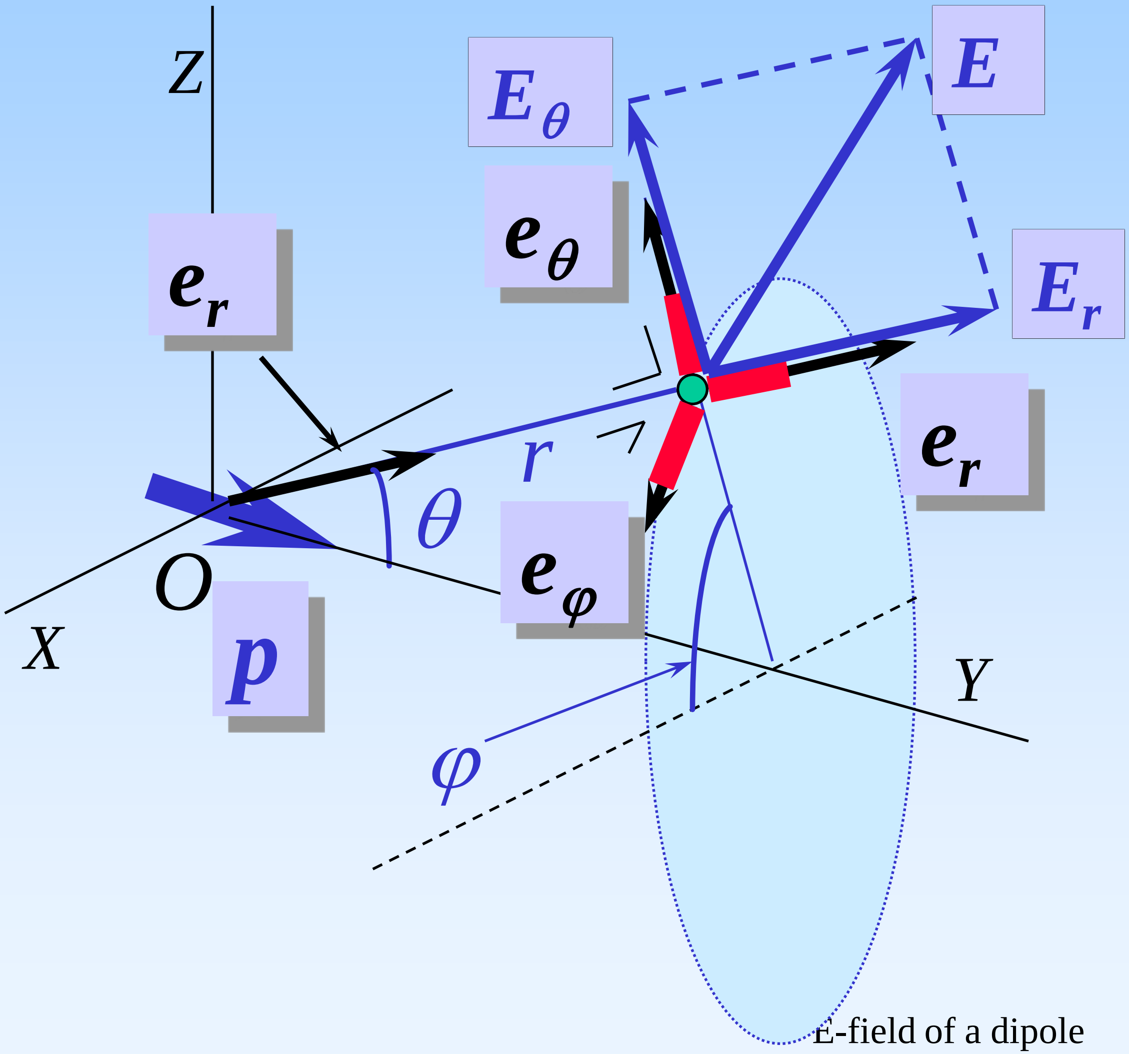
here $u=r$, $v=\theta$,$w=\varphi$

$$\partial s_v = r\sin\theta\,\partial\varphi$$

with $\quad V(r,\theta) = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

$$\boldsymbol{E}_\varphi = -\frac{\partial V}{r\sin\theta\,\partial\varphi}\boldsymbol{e}_\varphi = 0\boldsymbol{e}_\varphi$$

Calculation of $\boldsymbol{E}$ (5)

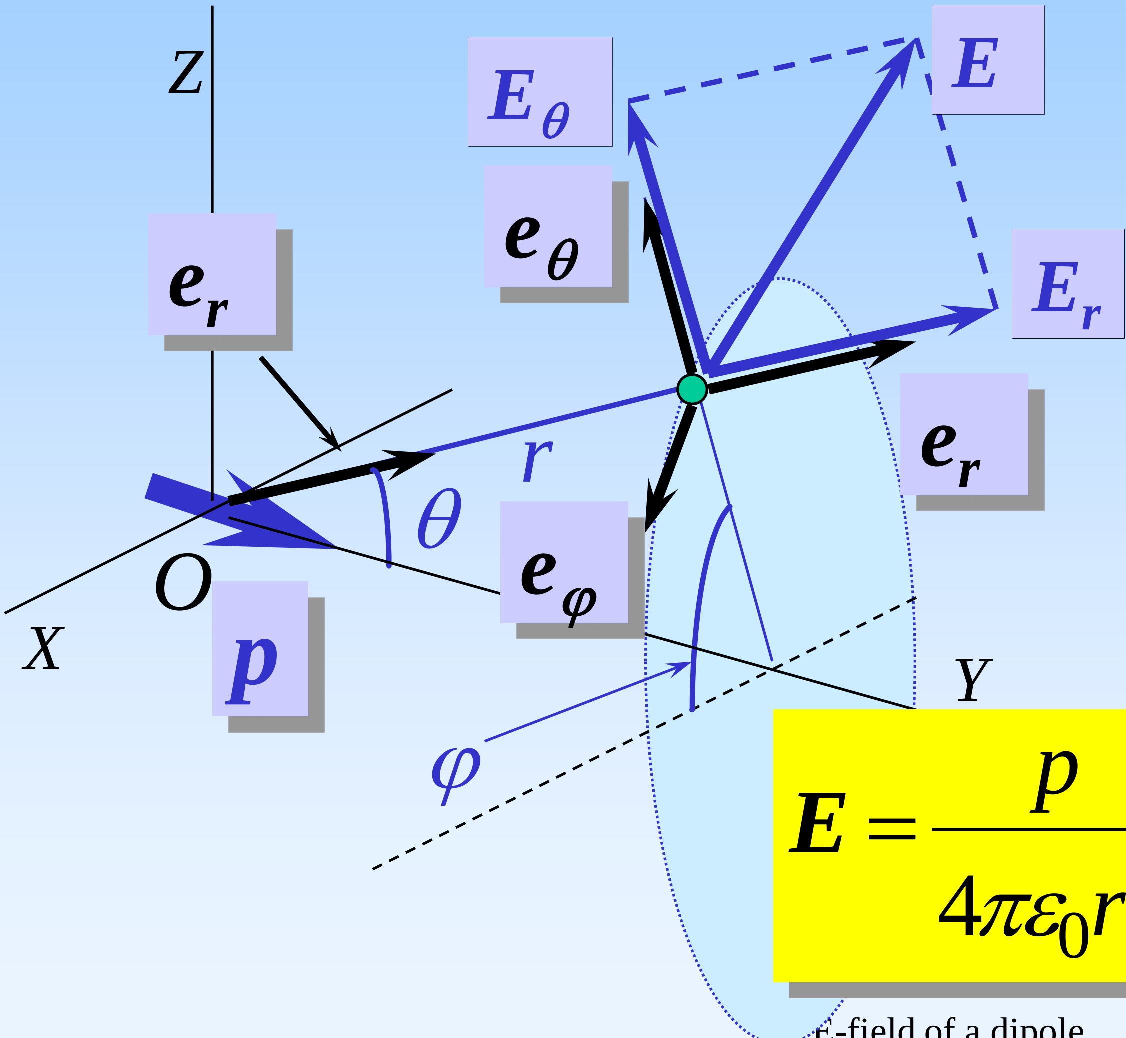


$$\boldsymbol{E}_r = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\boldsymbol{e}_r$$

$$\boldsymbol{E}_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\boldsymbol{e}_\theta$$

$$\boldsymbol{E}_\varphi = 0\boldsymbol{e}_\varphi$$

Conclusions (1)

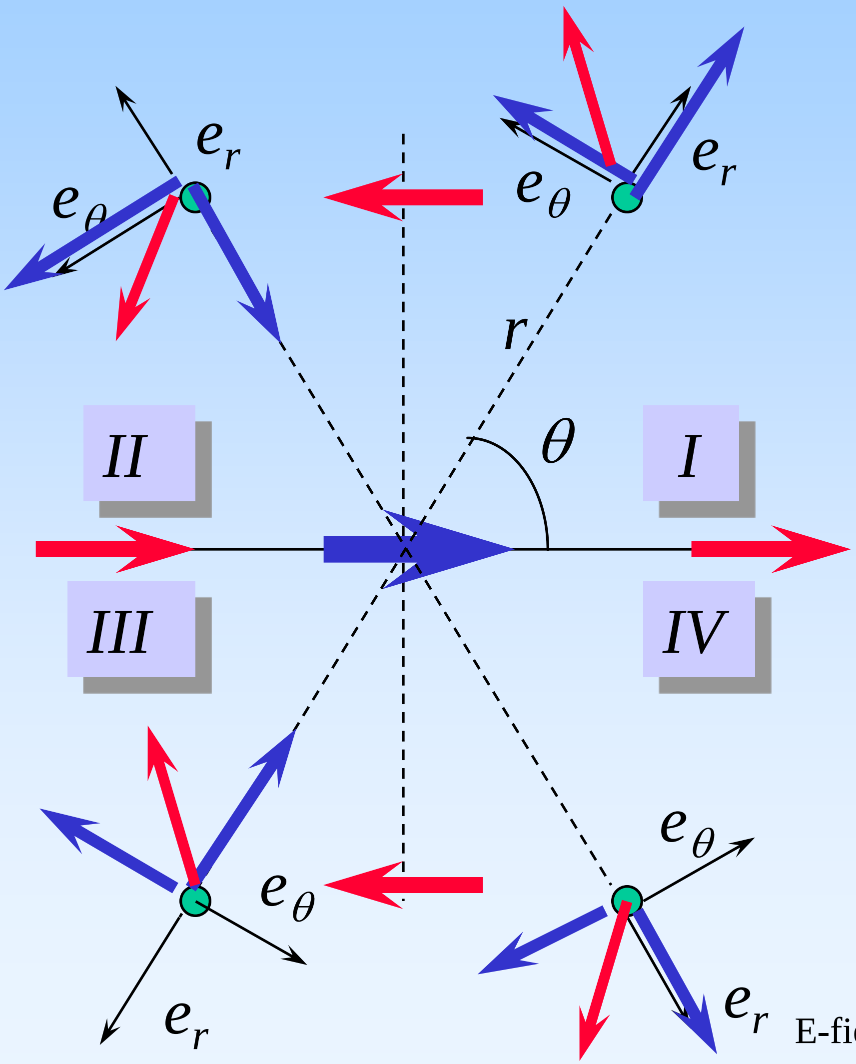


$$V(r,\theta) = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$V(r,\theta) = \frac{\boldsymbol{p} \bullet \boldsymbol{e_r}}{4\pi\varepsilon_0 r^2}$$

$$\boldsymbol{E} = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\boldsymbol{e_r} + \sin\theta\,\boldsymbol{e_\theta}\right)$$

Conclusions (2)

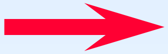


E -directions in the 4 quadrants:

$$\boldsymbol{E} = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\boldsymbol{e_r} + \sin\theta\,\boldsymbol{e_\theta}\right)$$

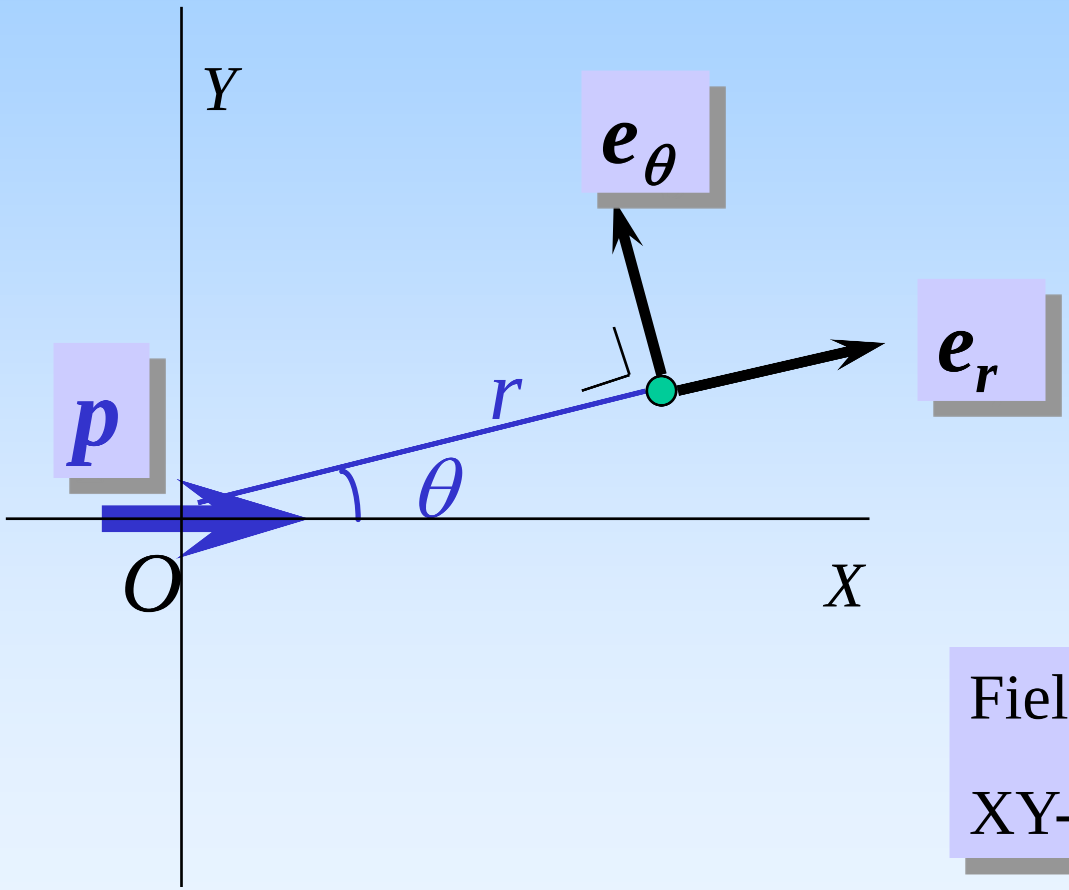
	I	II	III	IV
cos	+	-	-	+
sin	+	+	-	-

$\boldsymbol{E_r}$ and $\boldsymbol{E_\theta}$ -comp.



E

Field lines in XY-plane (1)

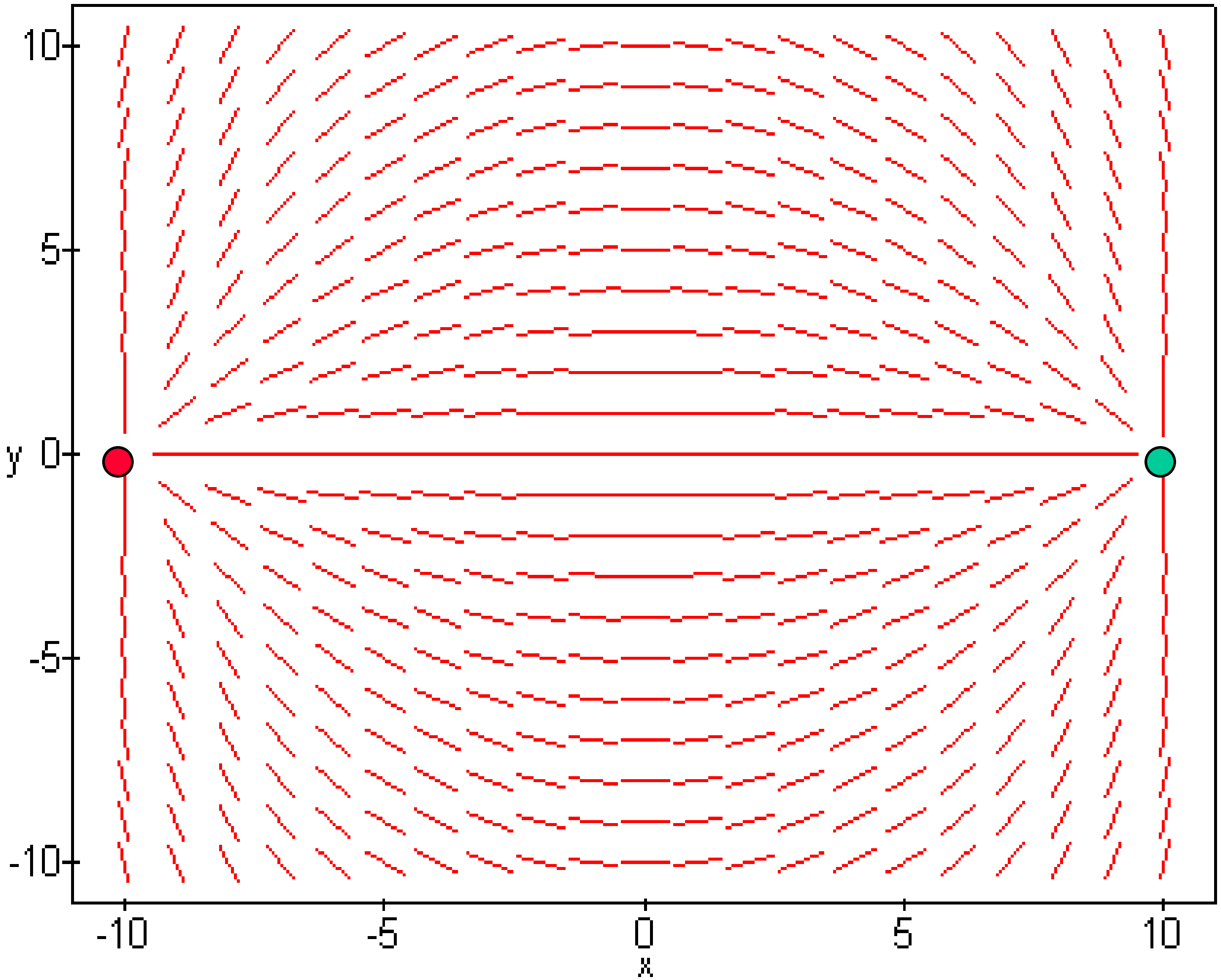


$$\boldsymbol{E}_r = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\boldsymbol{e}_r$$

$$\boldsymbol{E}_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\boldsymbol{e}_\theta$$

Field lines in XY-plane : $\dfrac{dy}{dx} = \dfrac{E_y}{E_x}$

Field lines in XY-plane (2)



Parameter: distance *a* between -Q and +Q

a = 20

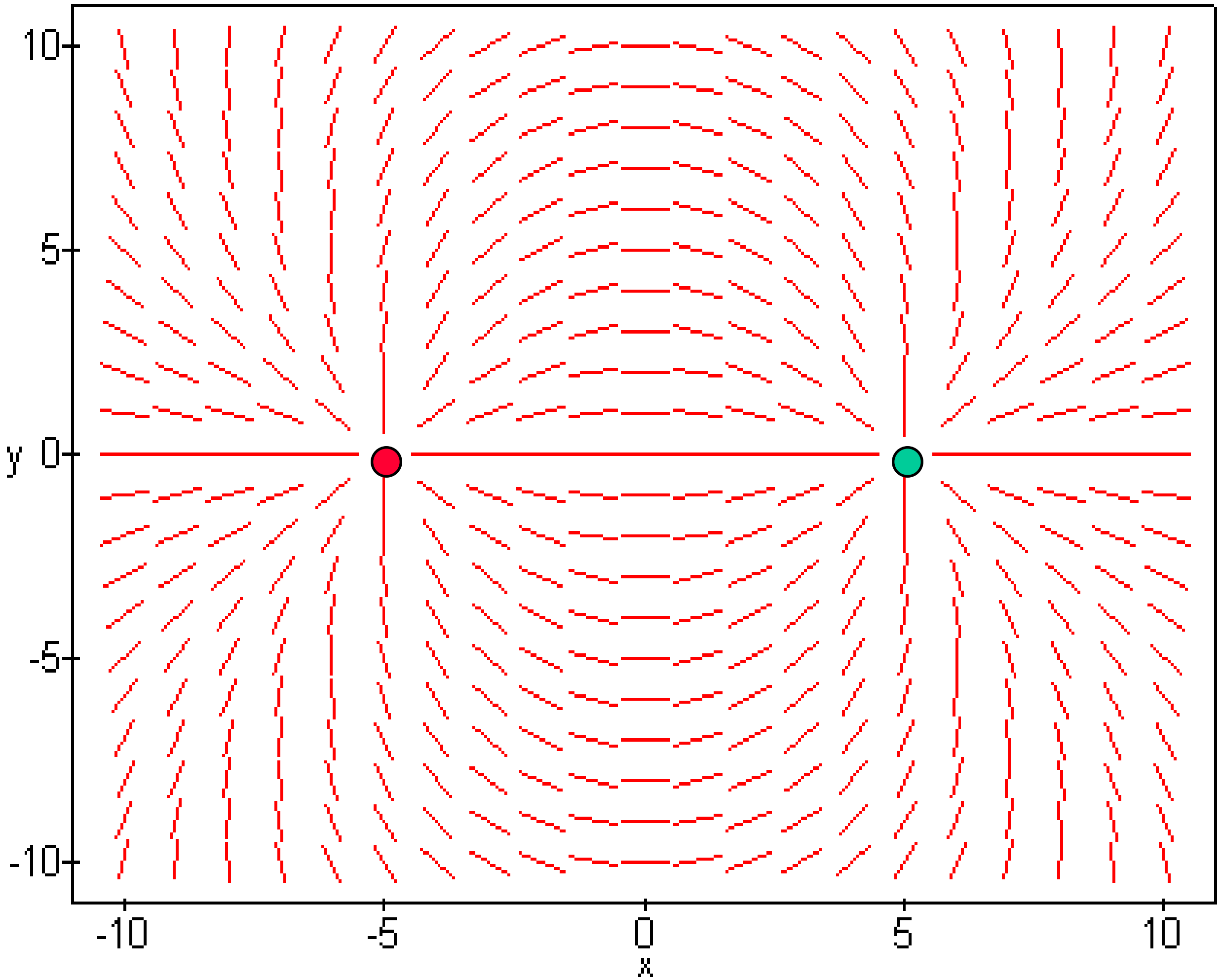
a = 10

a = 1

a = 0.1

far field

Field lines in XY-plane (3)



Parameter: distance *a* between -Q and +Q

a = 20

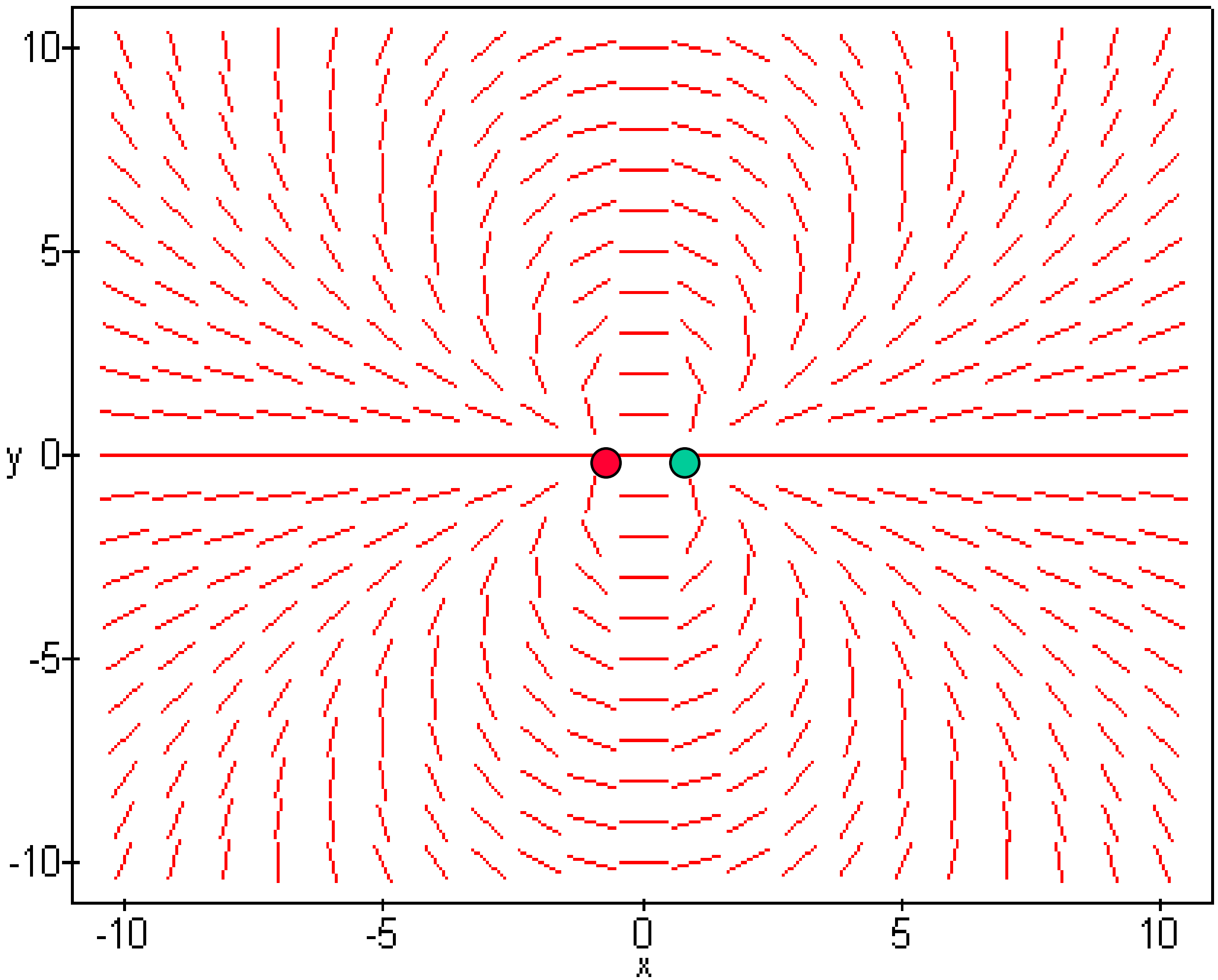
a = 10

a = 1

a = 0.1

far field

Field lines in XY-plane (4)



Parameter: distance *a* between -Q and +Q

a = 20

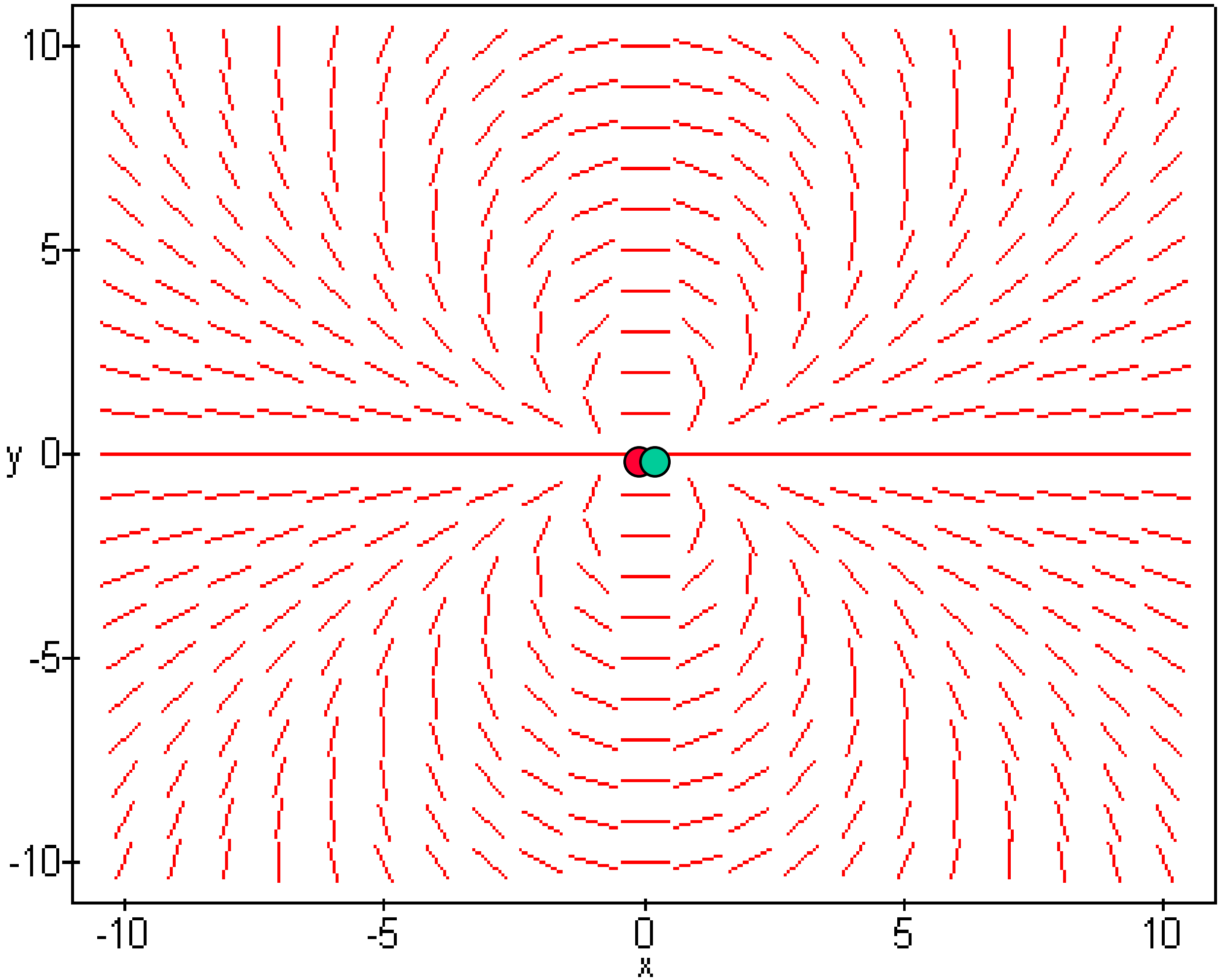
a = 10

a = 1

a = 0.1

far field

Field lines in XY-plane (5)



Parameter: distance *a* between -Q and +Q

a = 20

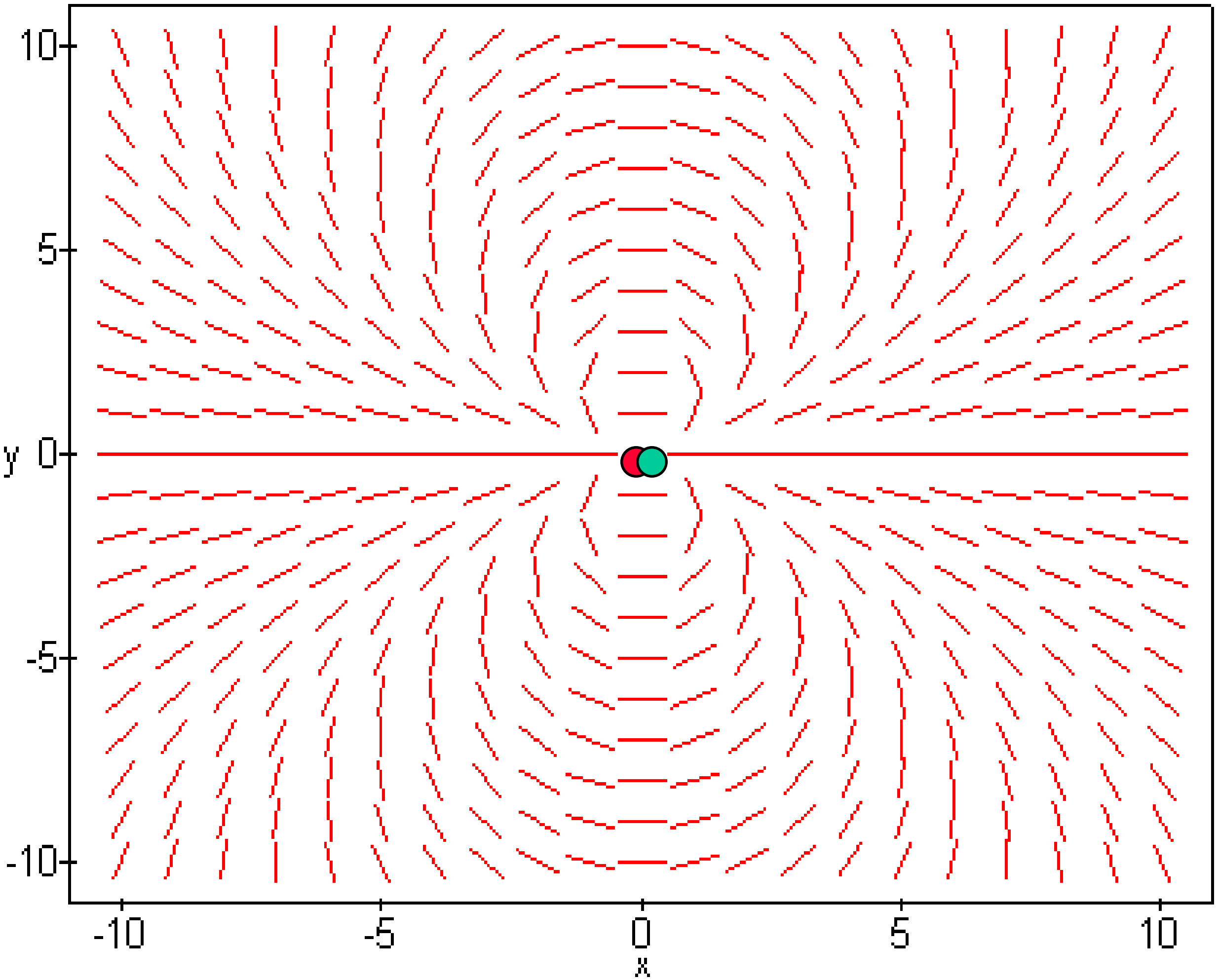
a = 10

a = 1

_a_ = 0.1

far field

Field lines in XY-plane (6)



Parameter: distance *a* between -Q and +Q

a = 20

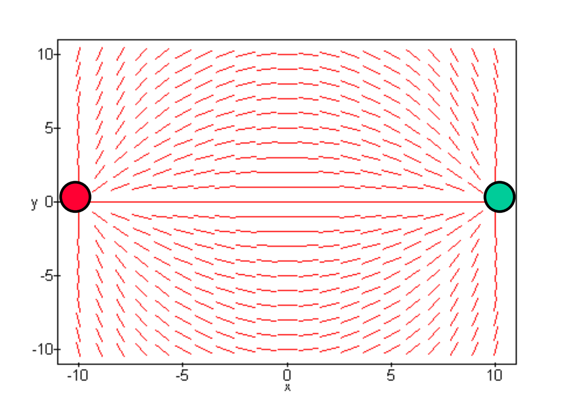
a = 10

a = 1

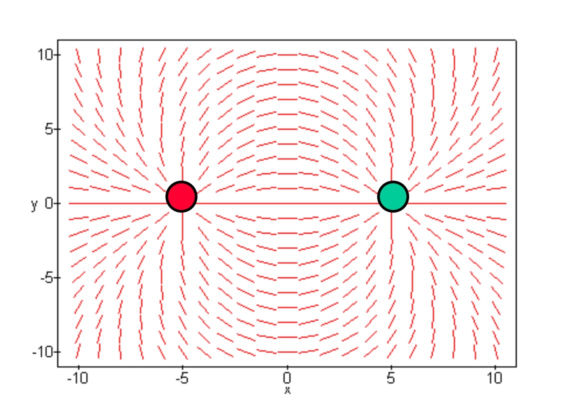
a = 0.1

far field

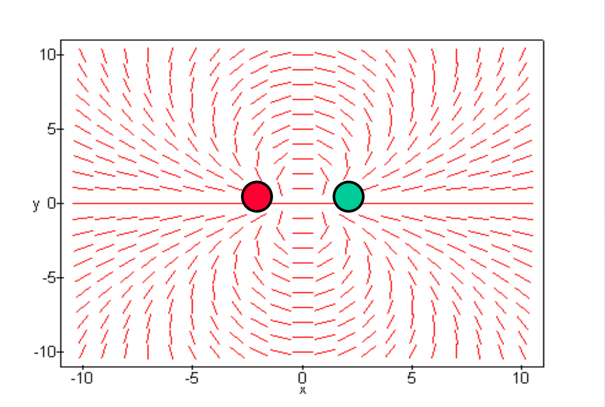
Field lines in XY-plane (7)



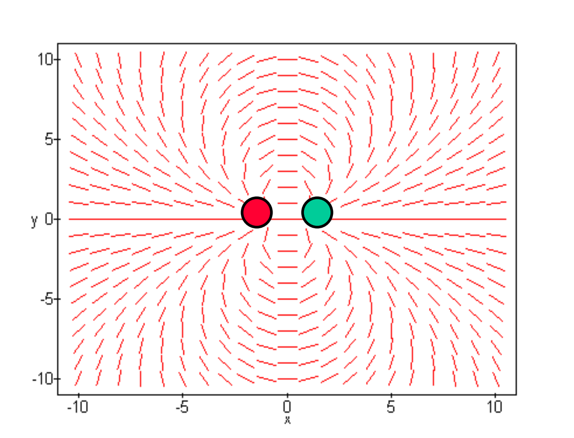
$a = 20$



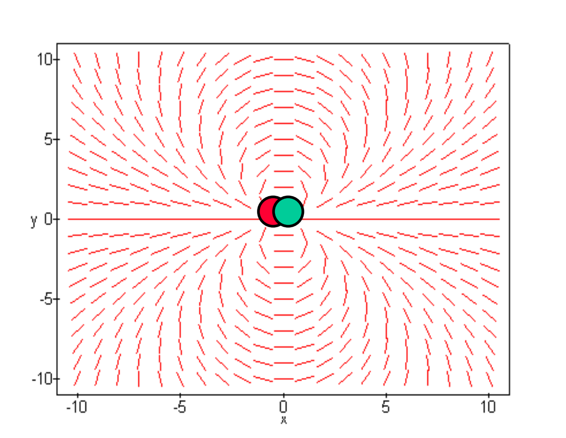
$a = 10$



$a = 1$



$a = 0.1$



far field

Parameter: distance a between -Q and +Q

the end