Filling a capacitor with a dielectric II: Partial filling

2nd version

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Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

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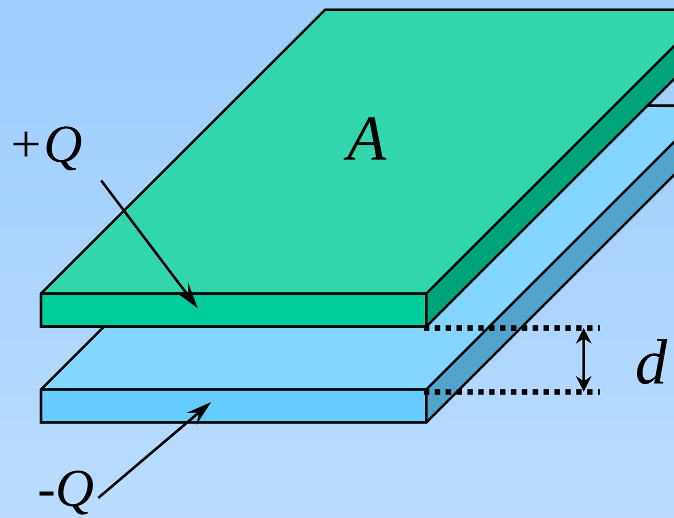
Partial filling of a capacitor (1)



<u>Available:</u> <u>Flat capacitor</u> :
= surface area A ,
= distance of plates d ,
= empty ($\varepsilon_r = 1$) .

<u>Assume:</u> initially, plates are charged with $+Q$, $-Q$.

$\sigma = Q/A$: charge density

$\boldsymbol{E}$ = electric field strength

$\boldsymbol{D}$ = dielectric displacement

ΔV = potential difference

C = capacitance

<u>Question:</u> What will happen with

Q , σ, $\boldsymbol{E}$, $\boldsymbol{D}$, ΔV and C

upon **PARTIALLY** filling the capacitor with dielectric material (with $\varepsilon_r > 1$) ???

Partial filling of a capacitor (2)

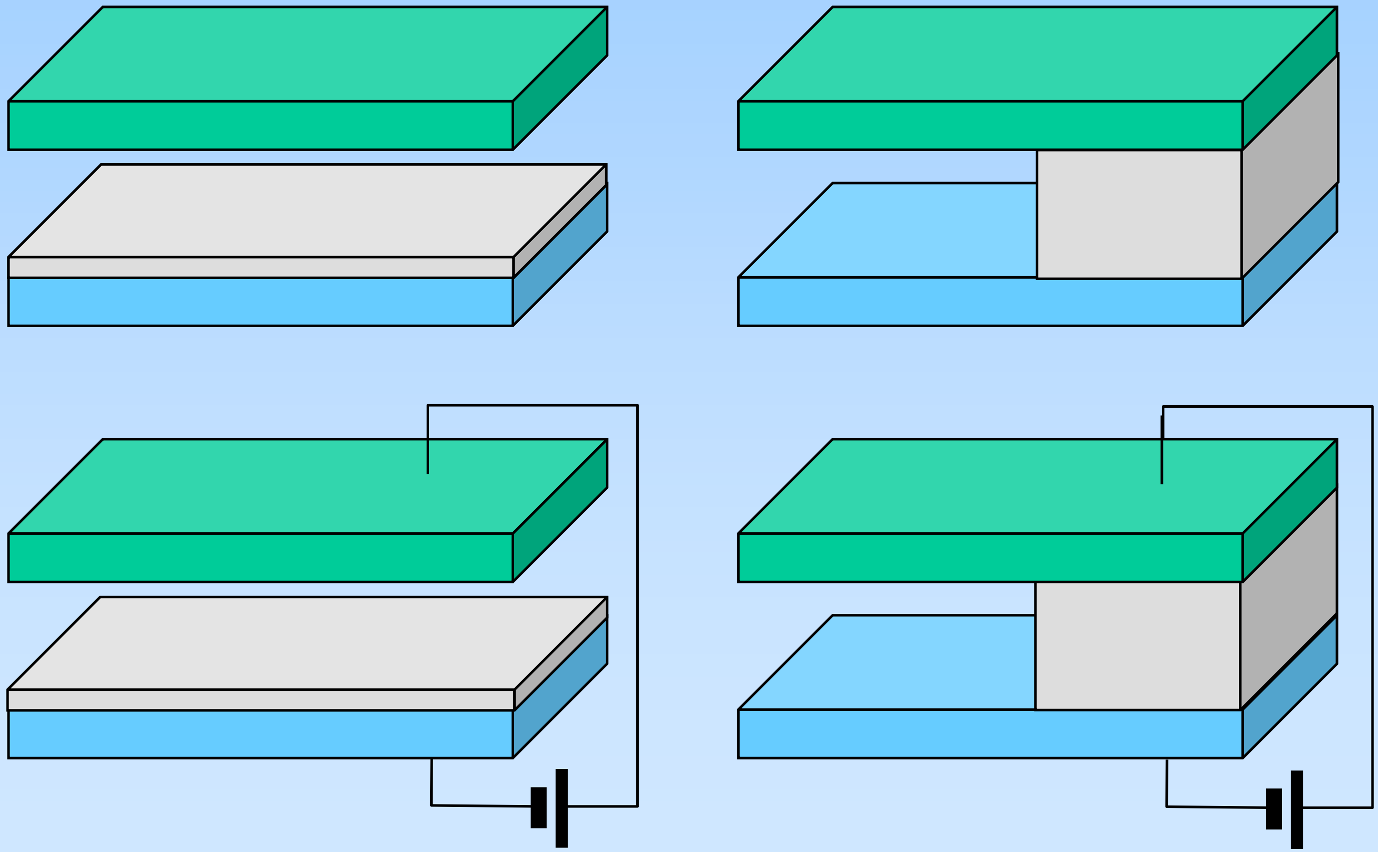
Options:

A. Series

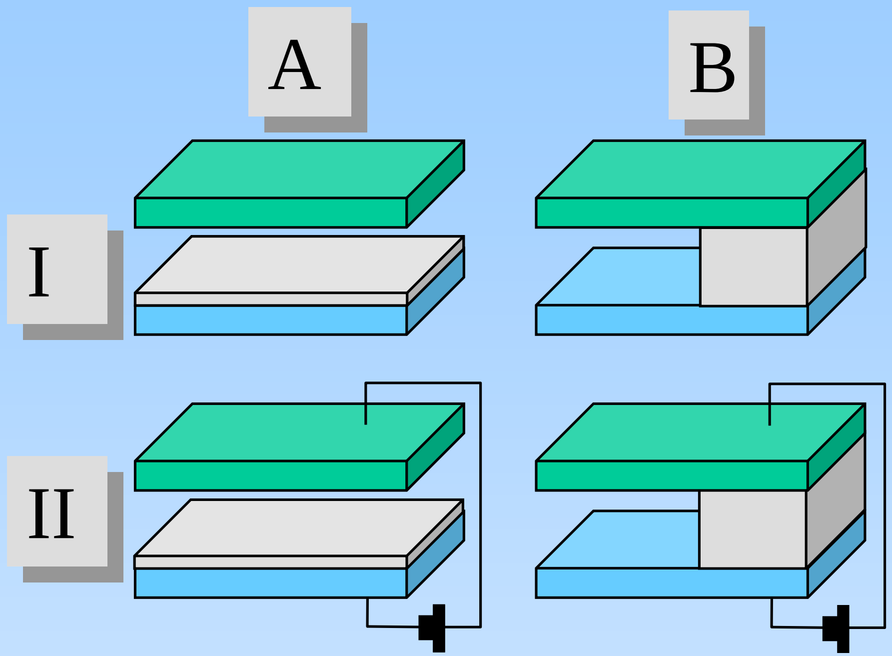
B. Parallel

I. Free

II. Connected to battery



Partial filling of a capacitor (3)



<u>Assume</u> :

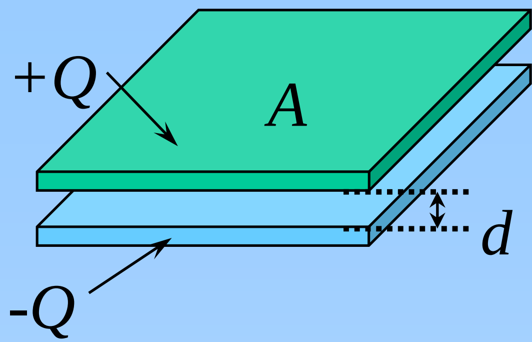
no “edge effects” :

d << plate dimensions

<u>Consequences</u> :

- no ***E***-field leakage
- planar symmetry everywhere
- Gauss’ Law is applicable
- straight field lines (***E*** and ***D***)

<u>Suppose</u>:
- when empty: Q_0 , V_0 ..... etc.
- when filled: Q', V' ..... etc.
- filling for 1/3 of volume,
- with $\varepsilon_r = 5$



Relations

for an ideal flat capacitor

Gauss:

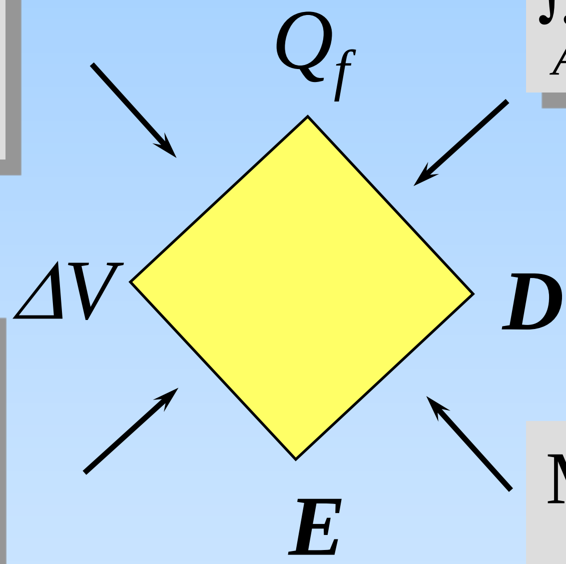
$$\oiint_A \mathbf{D} \bullet \mathbf{dA} = Q_f = \iiint_V \rho_f \, dV$$

$$Q_f = D.A = \sigma_f .A$$

Q_f = free charge

Capacitance:

$$C = Q_f / \Delta V$$



Potential:

$$\Delta V = \int_{path} \mathbf{E} \bullet \mathbf{dl}$$

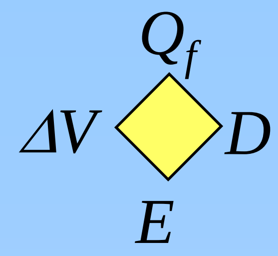
$$\Delta V = E.d$$

Material constants:

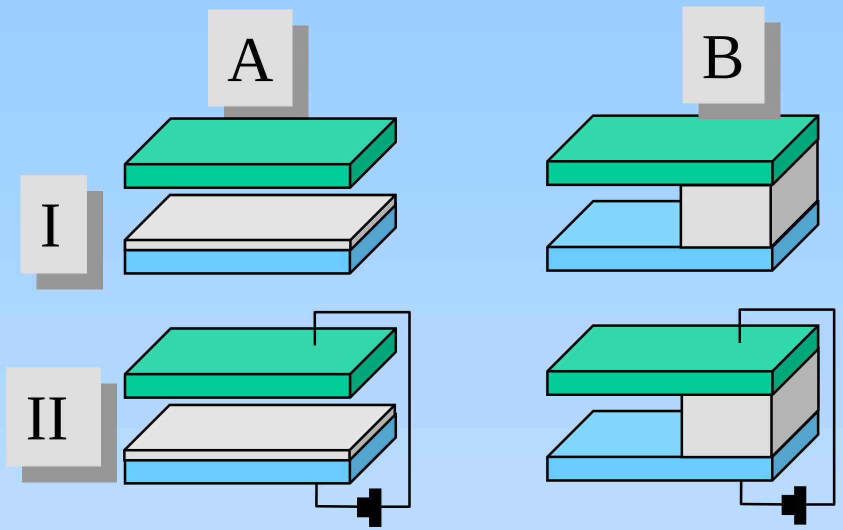
$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E}$$

Capacitance: $C = \varepsilon_0 \varepsilon_r A/d$

What to expect ??



$Q_f = D.A$
$D = \varepsilon_0 \varepsilon_r E$
$\Delta V = E.d$
$C = Q_f / \Delta V$



A.I and B.I: total <u>charge</u> unchanged

A.II and B.II: total <u>potential</u> unchanged

A.I and A.II:
potentials in <u>series</u>

B.I and B.II:
potentials <u>parallel</u>

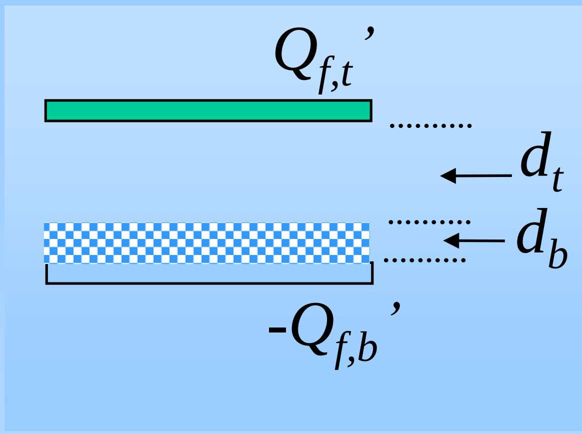
= equal ; ↑ up ; ↓ down

<..> = average
over both parts

I: Q_f = D = $\langle E \rangle$ ↓ ΔV ↓ C ↑

II: ΔV = E = $\langle D \rangle$ ↑ $\langle Q_f \rangle$ ↑ C ↑

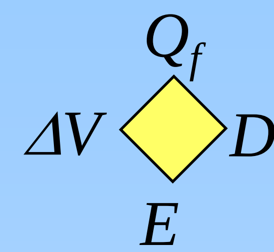
A.I. Horizontal filling, not connected



d_b , d_t : bottom and top layer

Fill: $d_b = d_0/3$ with $\varepsilon_r = 5$

$d_t = 2d_0/3$ with $\varepsilon_r = 1$



o = old (empty)

t = top

b = bottom

} total

$Q_f = D.A$

$D = \varepsilon_0 \varepsilon_r E$

$\Delta V = E.d$

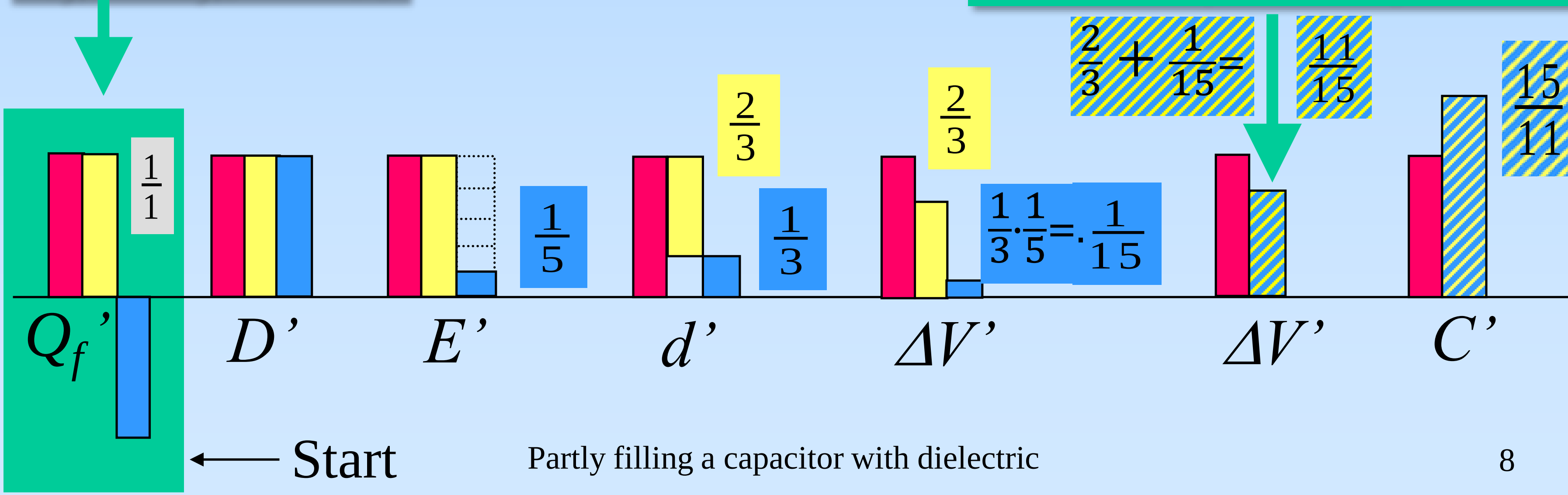
$C = Q_f / \Delta V$

Where to start ??

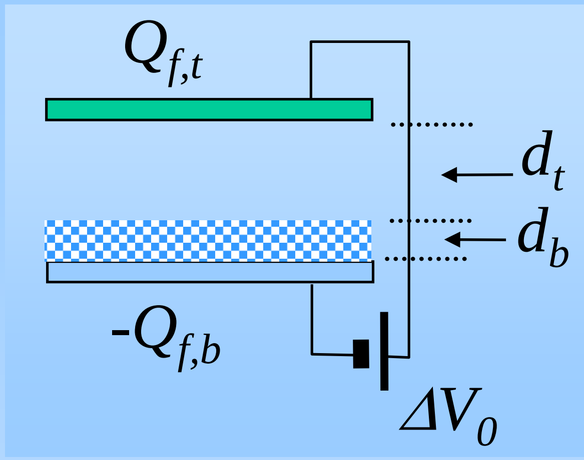
Q_f remains constant

$Q_{f,t}' = Q_{f,b}' = Q_0$

$\Delta V' = \Delta V'_t + \Delta V'_b \neq \Delta V_0$



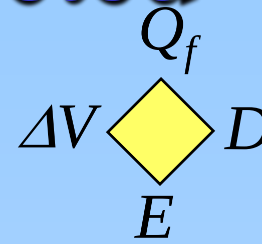
A.II. Horizontal filling, connected



Fill: $d_b = d_0/3$ with $\varepsilon_r = 5$
$d_t = 2d_0/3$ with $\varepsilon_r = 1$

o = old (empty)
t = top
b = bottom
} total

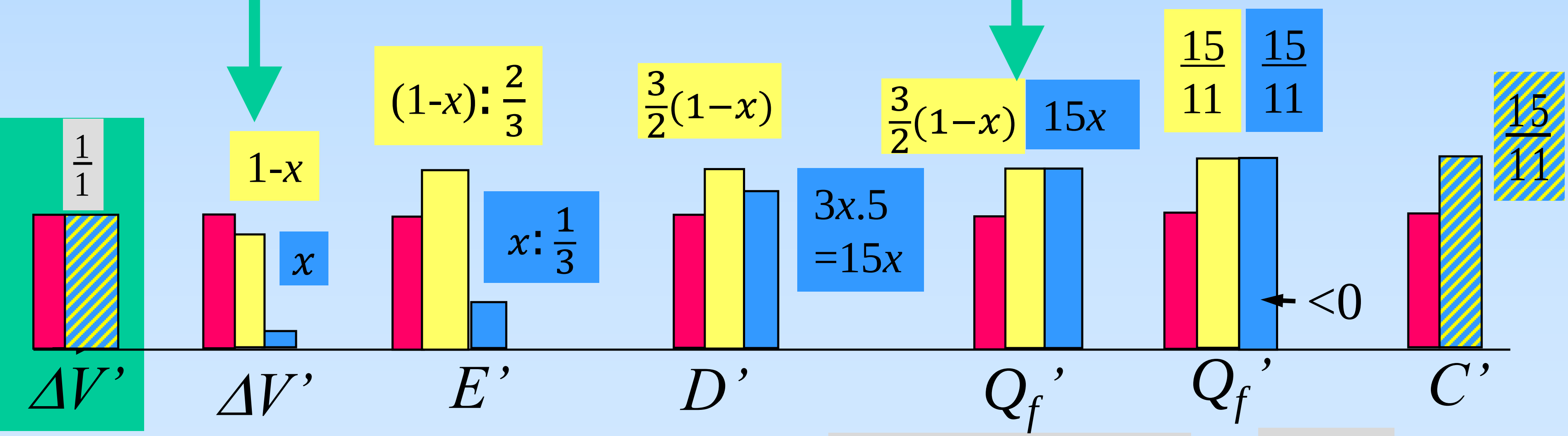
Start: ΔV remains constant



$Q_f = D.A$
$D = \varepsilon_0 \varepsilon_r E$
$\Delta V = E.d$
$C = Q_f / \Delta V$

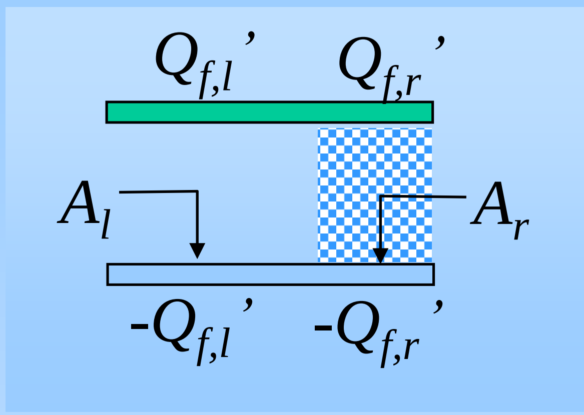
$\Delta V' = \Delta V'_t + \Delta V'_b = \Delta V_0$

$Q_{f,t}' = Q_{f,b}' \neq Q_0$



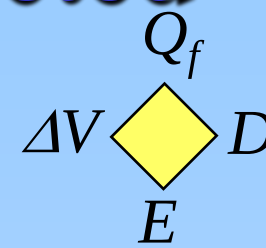
$\frac{3}{2}(1-x) = 15x$ $\quad x = \frac{1}{11}$

B.I. Vertical filling, not connected



Fill: right: $A_r = A_0/3$ with $\varepsilon_r = 5$

left: $A_l = 2A_0/3$ with $\varepsilon_r = 1$



- o = old (empty)
- l = left
- r = right

} total

Start: Q_f remains constant

$Q_f = D.A$

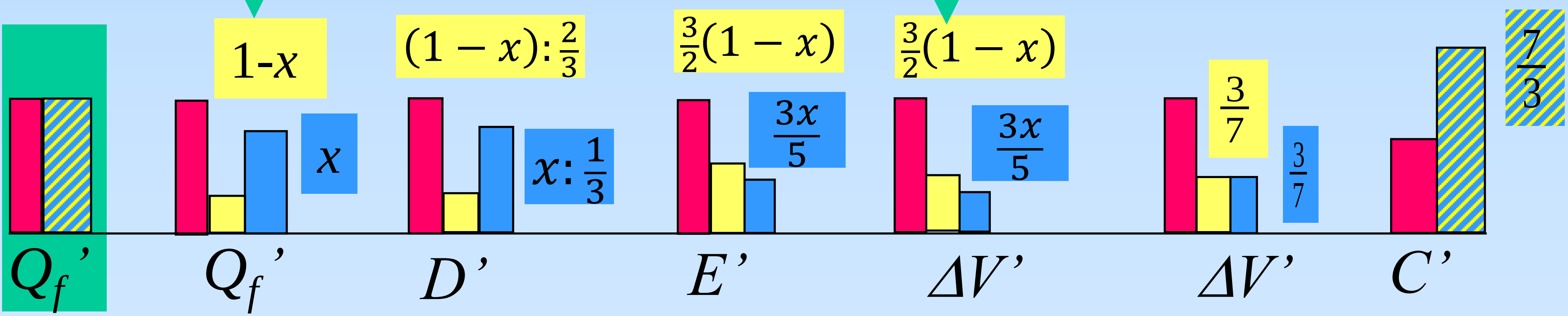
$D = \varepsilon_0 \varepsilon_r E$

$\Delta V = E.d$

$C = Q_f / \Delta V$

$Q_f' = Q_{f,l}' + Q_{f,r}' = Q_0$

$\Delta V_l' = \Delta V_r' \neq \Delta V_0$

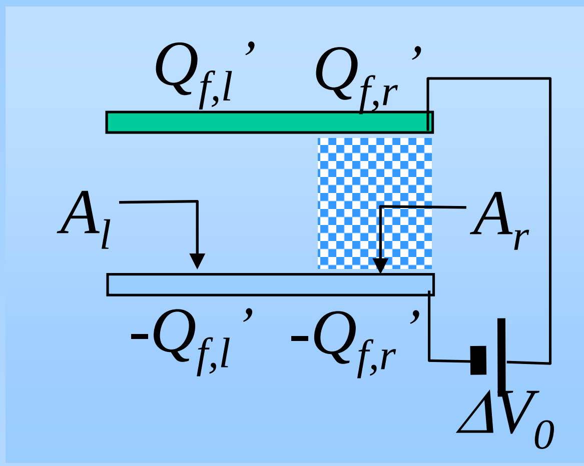


$\frac{3}{2}(1-x) = \frac{3x}{5}$

$x = \frac{5}{7}$

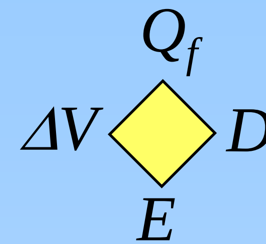
$\frac{3x}{5} = \frac{3}{7}$

B.II. Vertical filling, connected



Fill: $A_r = A_0/3$ with $\varepsilon_r = 5$
$A_l = 2A_0/3$ with $\varepsilon_r = 1$

o = old (empty)
l = left
r = right
} total

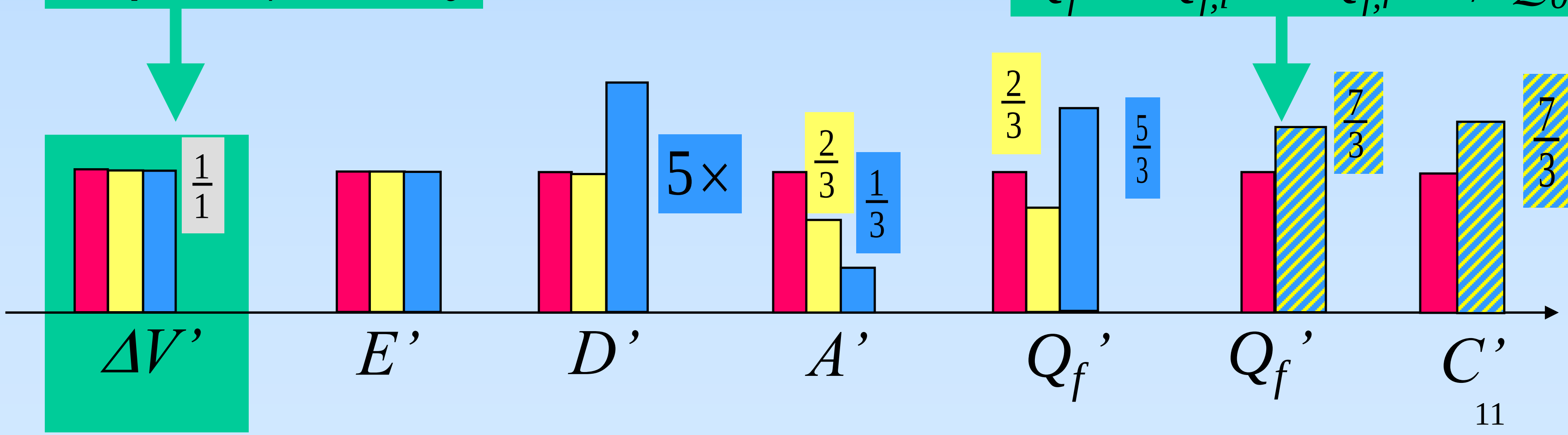


$Q_f = D.A$
$D = \varepsilon_0 \varepsilon_r E$
$\Delta V = E.d$
$C = Q_f / \Delta V$

Start: ΔV remains constant

$\Delta V_l' = \Delta V_r' = \Delta V_0$

$Q_f' = Q_{f,l}' + Q_{f,r}' \neq Q_0$

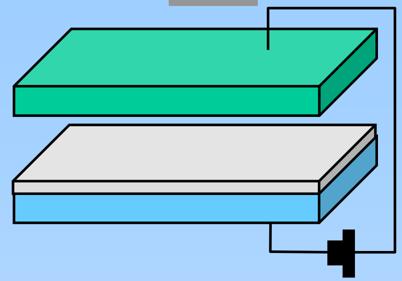


Overview: 1/3 filled, ε_r=5

I

II

A



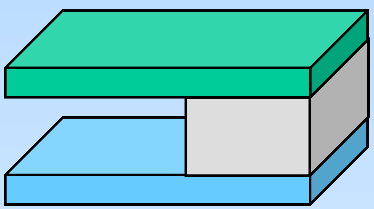
A.I and A.II:
potentials in series

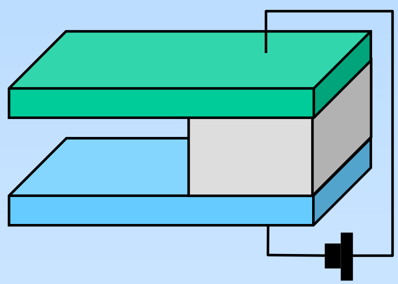
Q_f : unchanged
ΔV : 11/15 x

ΔV : unchanged
Q_f : 15/11 x

C : 15/11 x

B





B.I and B.II: potentials parallel

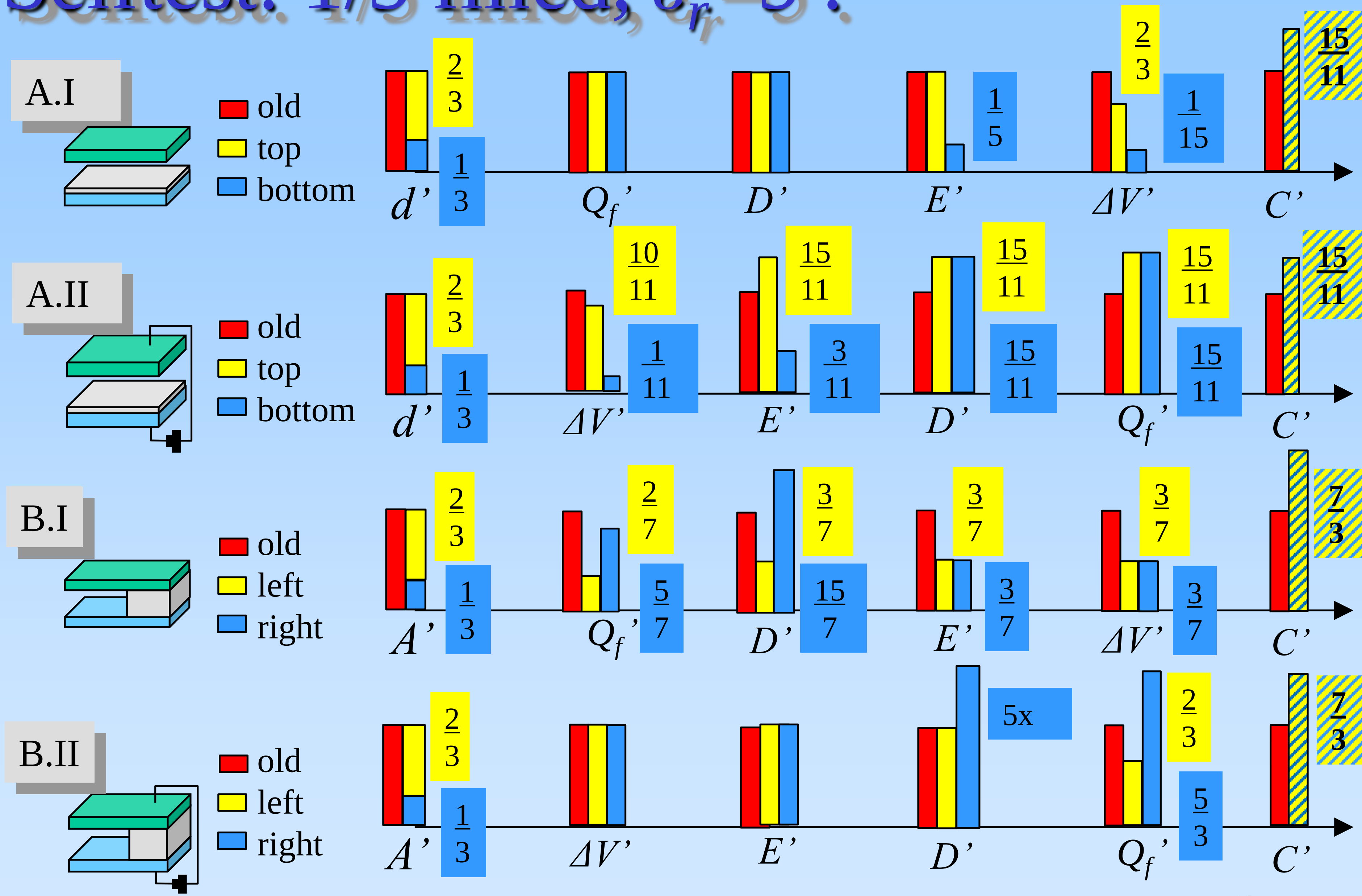
Q_f : unchanged
ΔV : 3/7 x

ΔV : unchanged
Q_f : 7/3 x

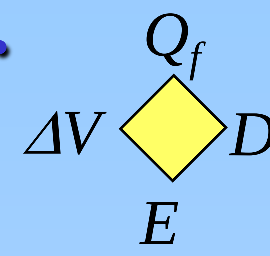
C : 7/3 x

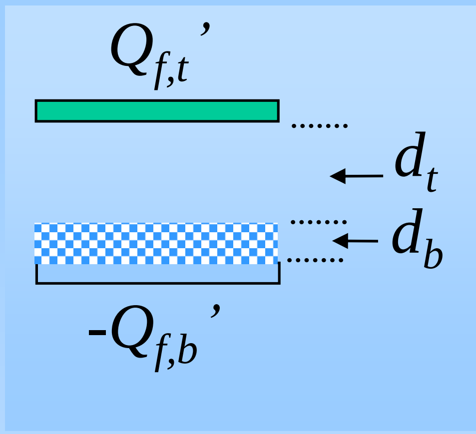
C does not depend on how the capacitor is used (intrinsic property)

Selftest: 1/3 filled, ε_r=5 :



Dielectric polarization P, for A.I.





d_b , d_t : bottom and top layer

Fill: $d_b = d_0/3$ with $\varepsilon_r = 5$

$d_t = 2d_0/3$ with $\varepsilon_r = 1$

o = old (empty)

t = top

b = bottom

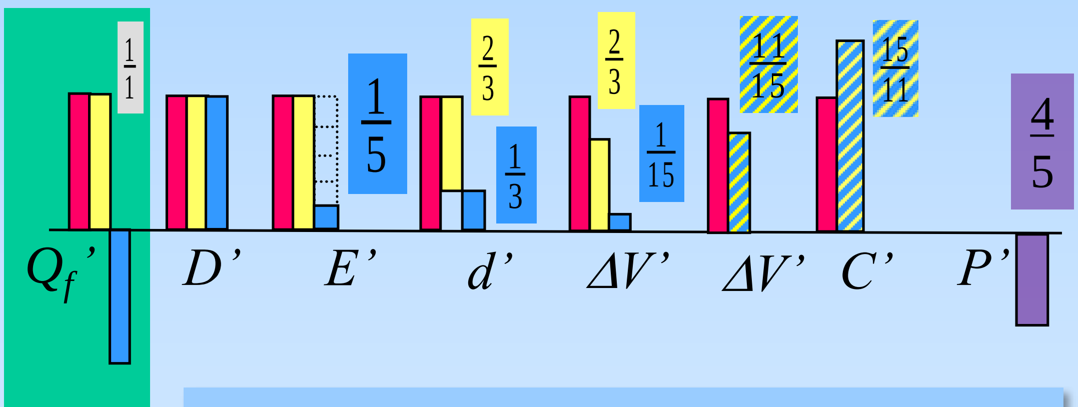
} total

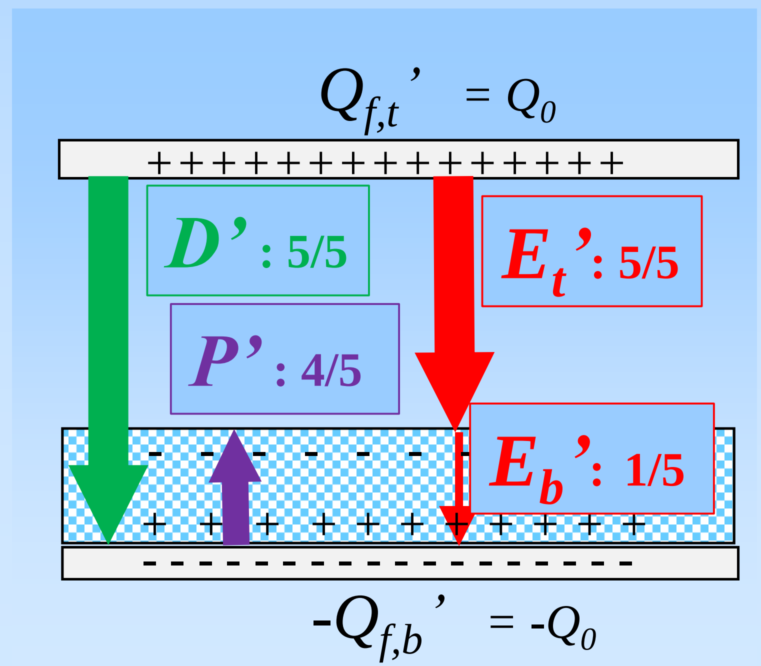
$Q_f = D.A$

$D = \varepsilon_0 \varepsilon_r E$

$\Delta V = E.d$

$C = Q_f / \Delta V$





Polarization: $\varepsilon_0 E = D - P$

$P = (\varepsilon_r - 1)\, \varepsilon_0 E = \chi_e\, \varepsilon_0 E$; E = total field

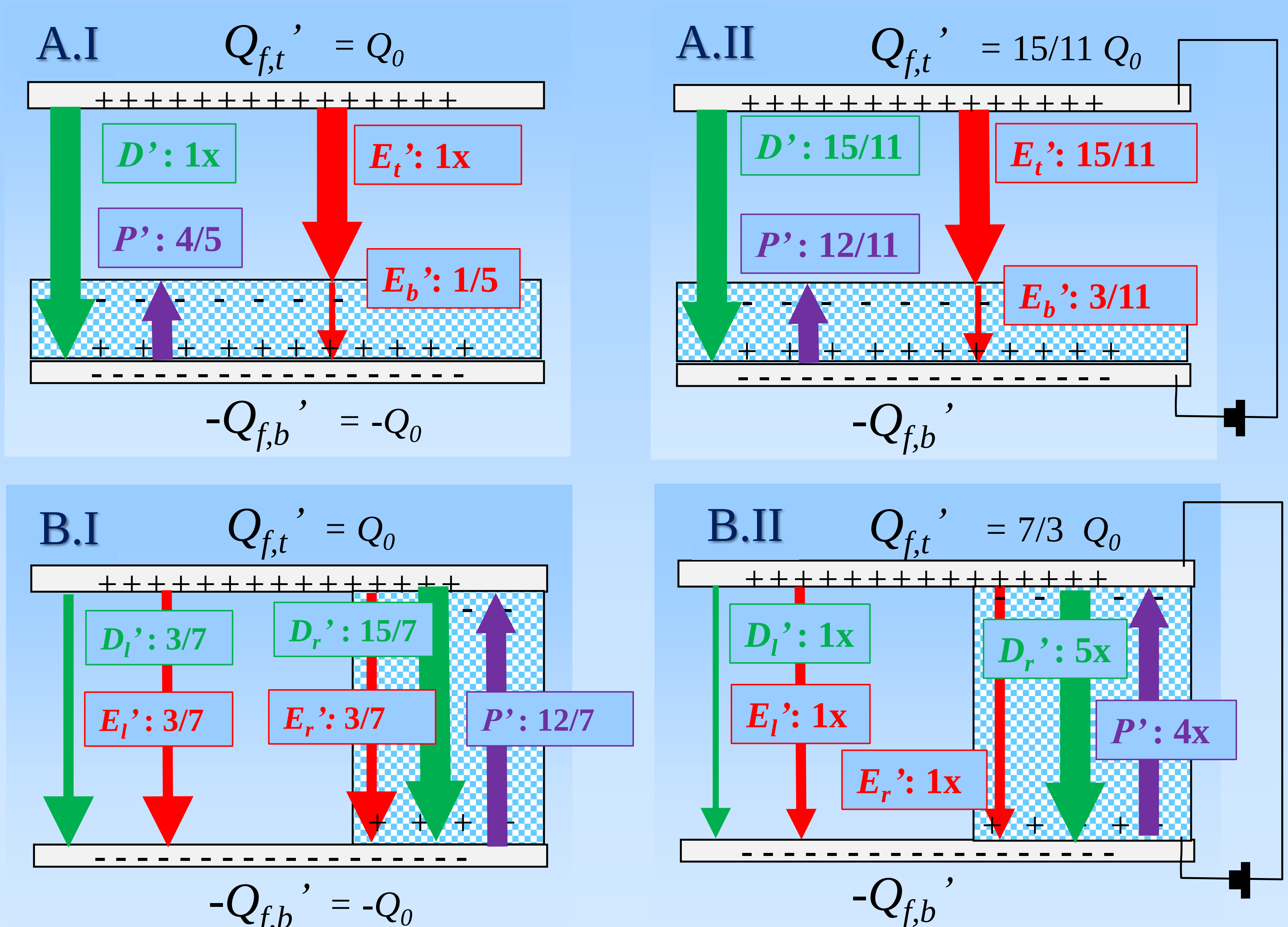
$\chi_e = \varepsilon_r - 1$ = electric susceptibility

Dielectric polarization P

Comparison with empty situation

$Q_f = D.A$
$D = \varepsilon_0 \varepsilon_r E$
$\Delta V = E.d$

$\varepsilon_0 E = D - P$



Homework 1

In the example, the capacitor was filled for 1/3 of the volume, with $\varepsilon_r = 5$.

Now suppose that the filling was: for a volume fraction p, with $\varepsilon_r > 1$.

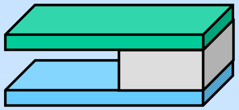
Show that the capacitance changes with a factor F:

$$F = \left[1 - p + \frac{p}{\varepsilon_r}\right]^{-1}$$

for SERIES

$$F = 1 - p + \varepsilon_r . p$$

for PARALLEL

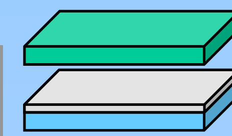


Note: with ε_r >1: for all p with $0 \leq p \leq 1$: $1 \leq F(p) \leq \varepsilon_r$.

Homework 1

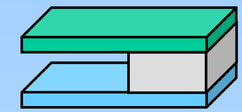
$$F = \left[1 - p + \frac{p}{\varepsilon_r}\right]^{-1}$$

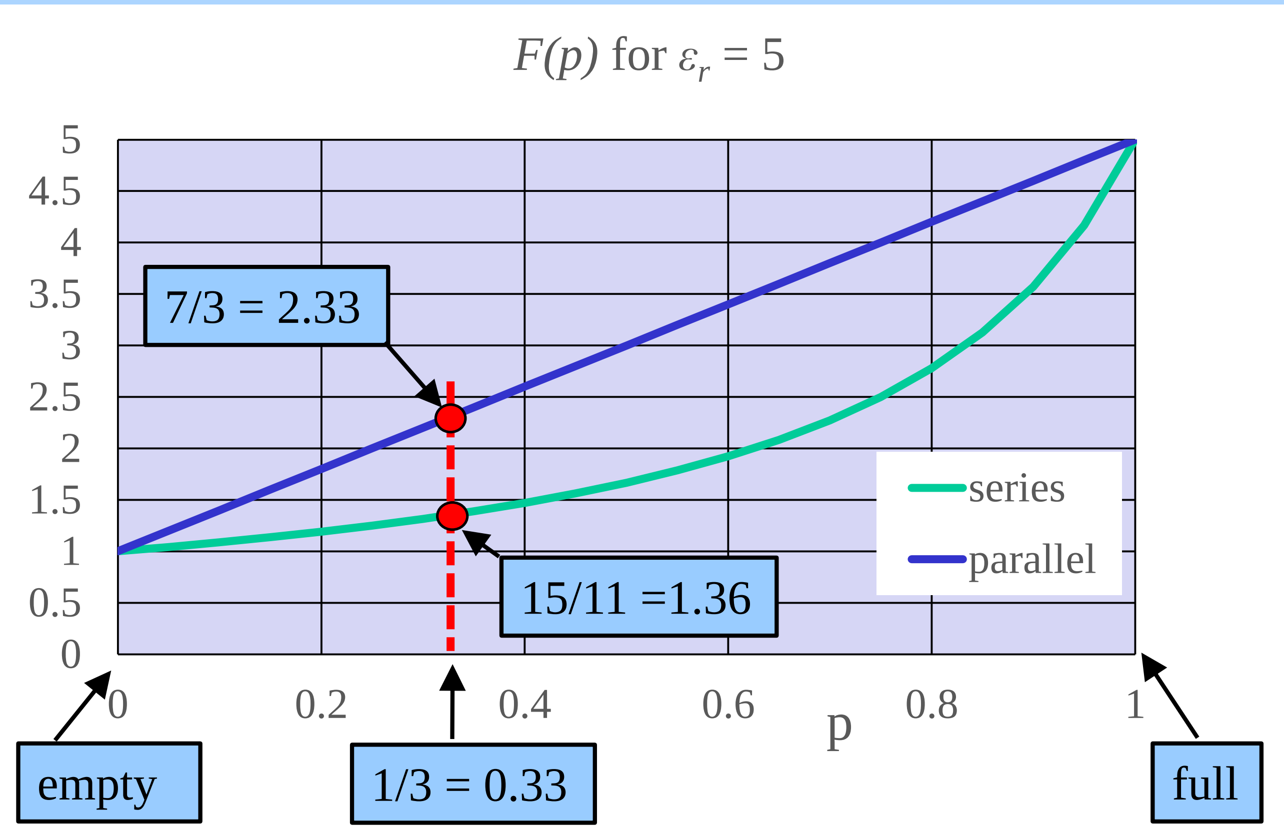
for SERIES



$$F = 1 - p + \varepsilon_r . p$$

for PARALLEL





Example:
for p = 1/3
and ε_r = 5 :

series: F = 15/11
parallel: F = 7/3

Homework 2

In the example, the capacitor was filled for 1/3 of the volume, with $\varepsilon_r = 5$.

Now suppose that the filling was: 2 dielectrics,

- for volume fraction p, with $\varepsilon_{r1} > 1$, and
- for volume fraction 1-p, with $\varepsilon_{r2} > 1$.

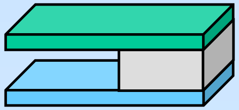
Show that the capacitance changes with a factor F:

$$F = \frac{\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r1}(1-p)+\varepsilon_{r2}p}$$

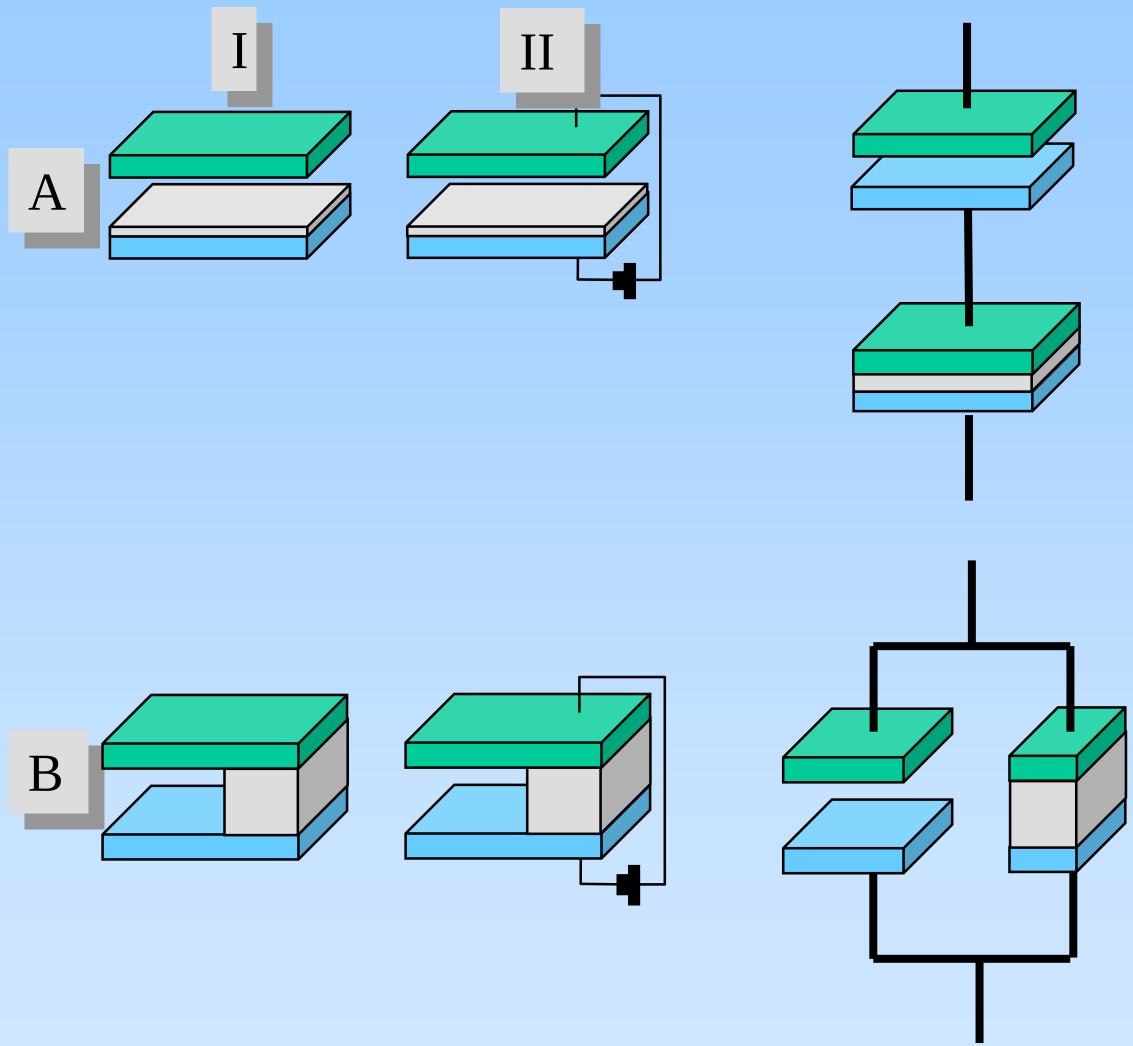
for SERIES

$$F = \varepsilon_{r2}(1-p) + \varepsilon_{r1} .p$$

for PARALLEL



Finally... Combination rules



A.I and A.II:
series

$$\frac{1}{C'} = \frac{1}{C_t} + \frac{1}{C_b}$$

B.I and B.II:

parallel

$$C' = C_l + C_r$$

the end