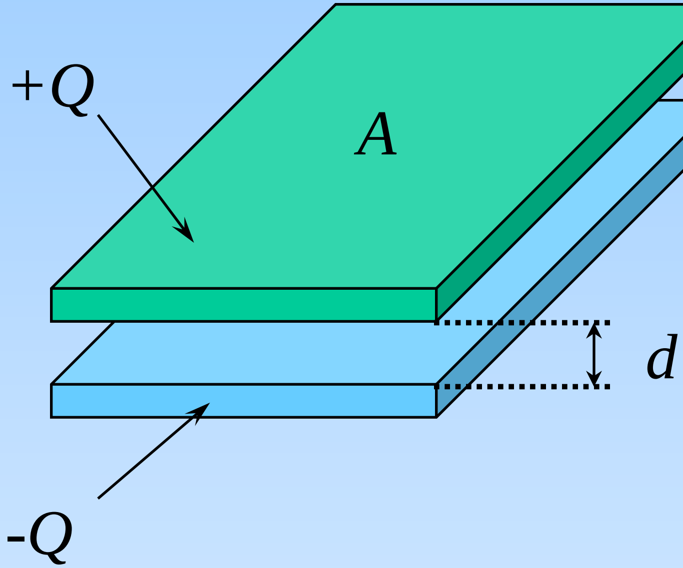
Filling a capacitor with a dielectric: I. Complete filling

Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

Filling a capacitor with dielectric

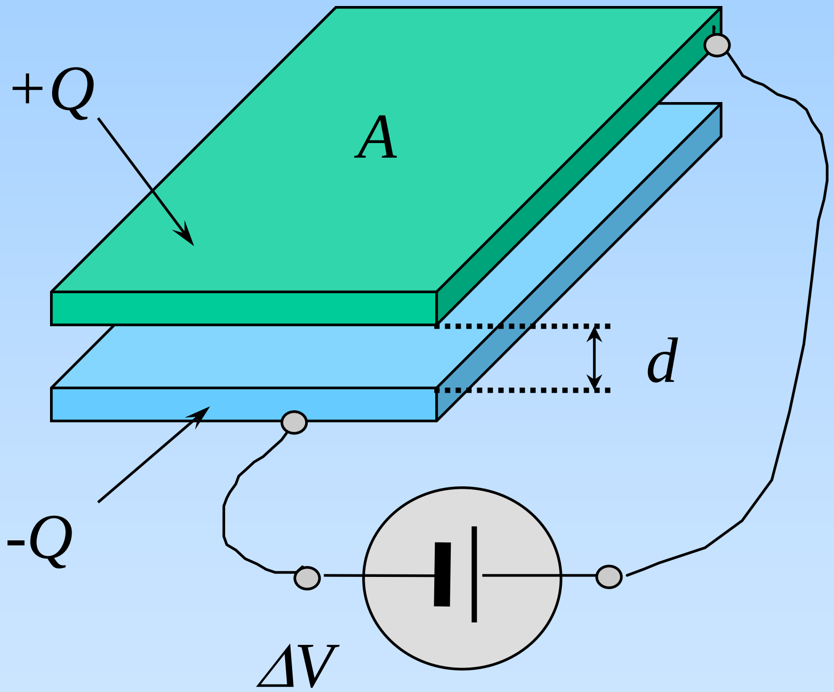


Available: Flat capacitor :
= surface area A ,
= distance of plates d ,
= empty ($\varepsilon_r = 1$) .

Assume: initially, plates are charged with $+Q$, $-Q$.

Question: What will happen with Q , σ, **E**, **D**, ΔV and C upon completely filling the capacitor with dielectric material (with $\varepsilon_r > 1$) ???

Analysis



Important : Does the capacitor remain connected to the battery that loaded it?

Case A : yes

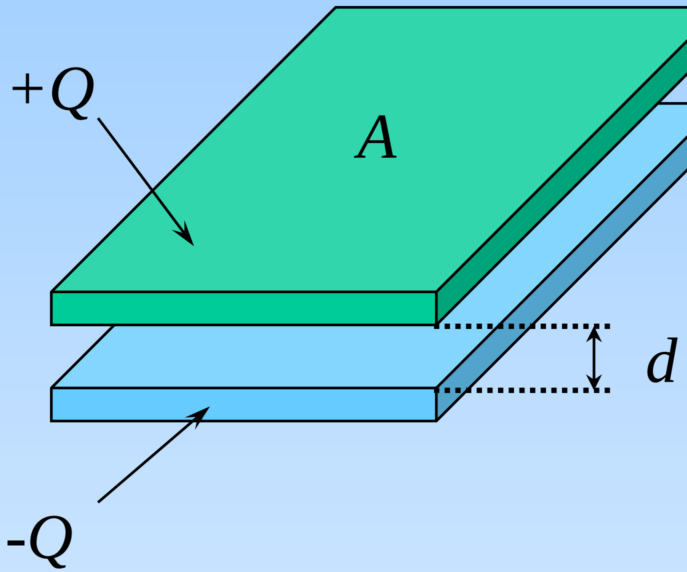
Case B : no

Consequences :

in A : voltage remains constant

in B : charges remain constant.

Approach (1)



<u>Assumption</u> :

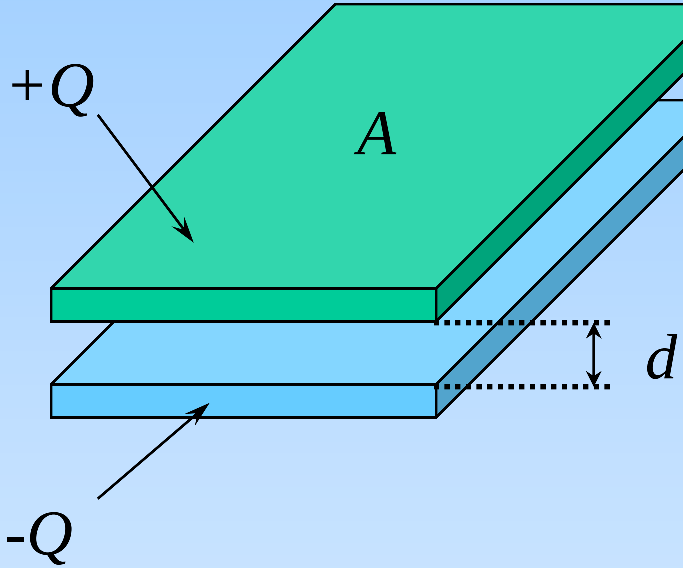
no "edge effects" :
d << plate dimensions

<u>Consequences</u> :

no ***E***-field leakage

planar symmetry everywhere :
Gauss' Law is applicable

Approach (2)



Relevant variables :

Q , σ, $\mathbf{E}$, $\mathbf{D}$, ΔV and C

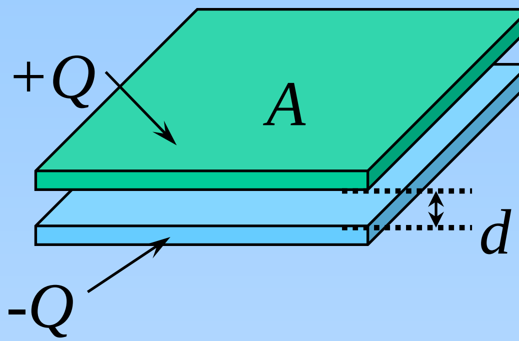
Material constants :

ε_0 , ε_r , A and d .

Task :

Establish the relations between the variables.

Approach (3)



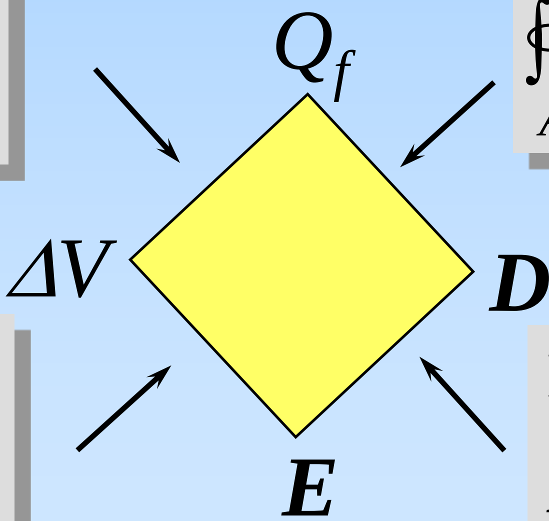
Relations between variables :

Gauss:

$$\oiint_A \boldsymbol{D} \bullet d\boldsymbol{A} = Q_f = \iiint_V \rho_f \, dV$$

Capacitance:

$$C = Q_f \,/\, \Delta V$$



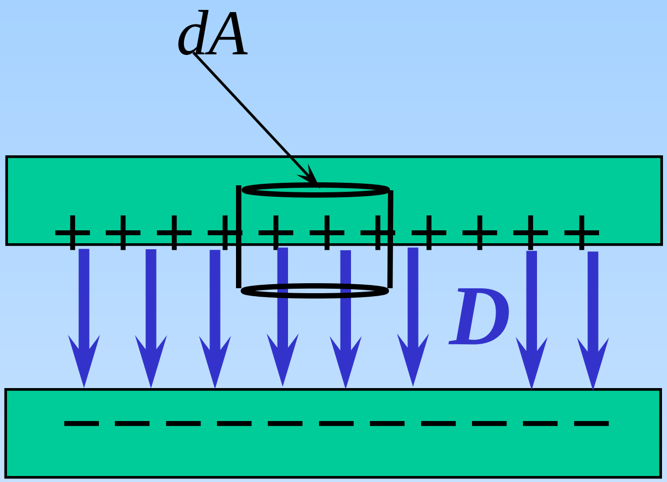
Potential:

$$\Delta V = \int_{path} \boldsymbol{E} \bullet d\boldsymbol{l}$$

Material constants:

$$\boldsymbol{D} = \varepsilon_0 \, \varepsilon_r \, \boldsymbol{E}$$

Calculations (1)



Gauss:

$$\underset{A}{\oiint} \boldsymbol{D} \bullet \boldsymbol{dA} = Q_f = \underset{V}{\iiint} \rho_f \, dV$$

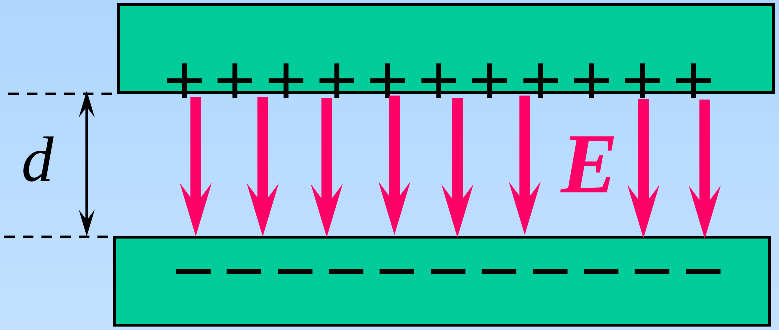
Take Gauss pill box,
top and bottom area = dA ,
enclosed charge = $\sigma_f . dA$

$$\underset{dA}{\oiint} \boldsymbol{D} \bullet \boldsymbol{dA} = D.dA$$

$\Rightarrow D = \sigma_f$

Calculations (2)

Potential : $\Delta V = \int_{path} \boldsymbol{E} \bullet \boldsymbol{dl}$



Take integration path from bottom to top, length *d*

$$\int_{bottom}^{top} \boldsymbol{E} \bullet \boldsymbol{dl} = E.d$$

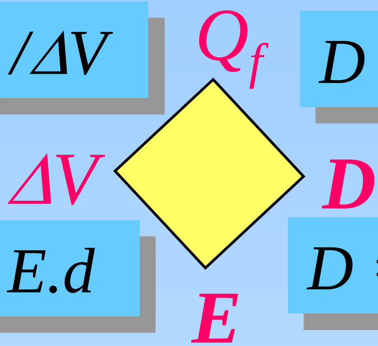
=> $\Delta V = E.d$

Calculations (3)

Relations for an ideal capacitance

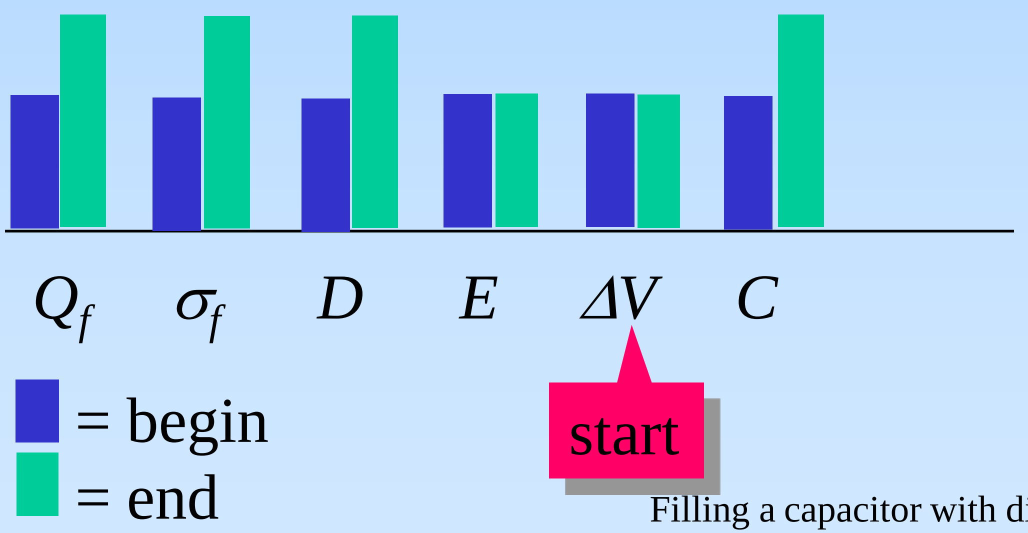
$C = Q_f / \Delta V$

$D = \sigma_f = Q_f / A$



$\Delta V = E.d$

$D = \varepsilon_0 \varepsilon_r E$



<u>Case A</u> : capacitor connected to battery

Where to start?????

ΔV remains constant !!

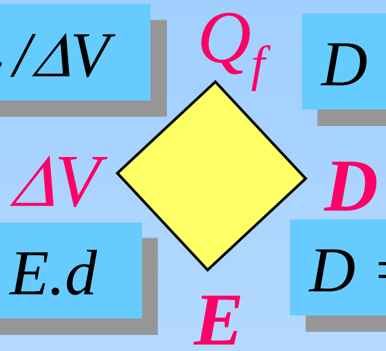
1. ΔV = const
2. E = const
3. D : ε_r x as large
4. σ_f : ε_r x as large
5. Q_f : ε_r x as large
6. C : ε_r x as <u>large</u>

Calculations (4)

Relations for an ideal capacitance

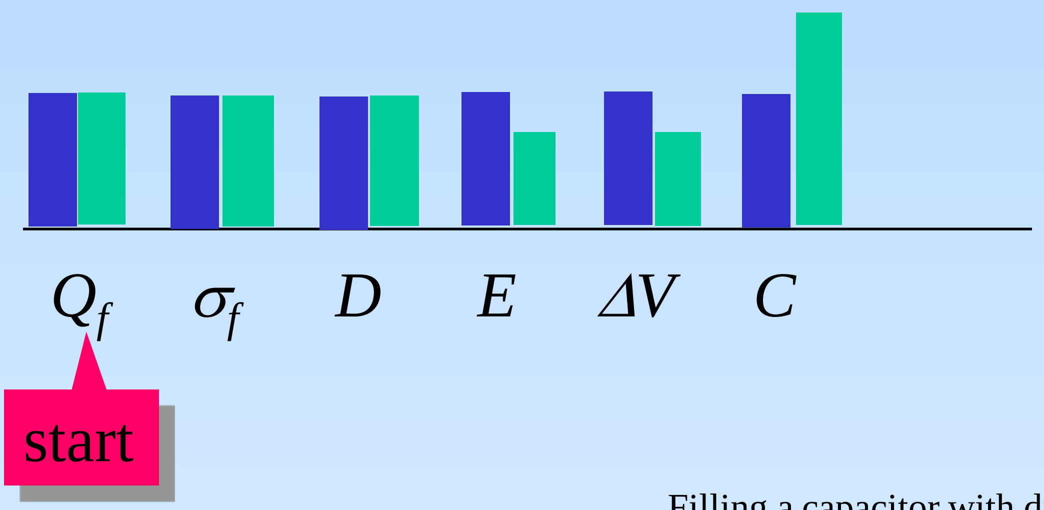
$C = Q_f / \Delta V$

$D = \sigma_f = Q_f / A$



$\Delta V = E.d$

$D = \varepsilon_0 \varepsilon_r E$



Case B : capacitor NOT connected to battery

Where to start?????

Q_f remains constant !!

1. Q_f = const
2. σ_f = const
3. D = const
4. E : ε_r x as small
5. ΔV : ε_r x as small
6. C : ε_r x as large

Conclusions

<u>if connected to battery</u>

- ΔV = const
- E = const
- D, σ_f , Q_f : ε_r x as large
- C : ε_r x as <u>large</u>

<u>if NOT connected</u>

- Q_f = const
- σ_f , D = const
- E , ΔV : ε_r x as small
- C : ε_r x as <u>large</u>

the end