Magnetic Field of a Long Wire

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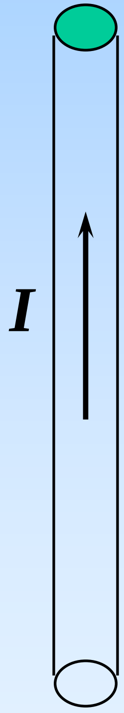
Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

B-field of a long wire



Available:

A thin wire, infinitely long, carrying a current *I*

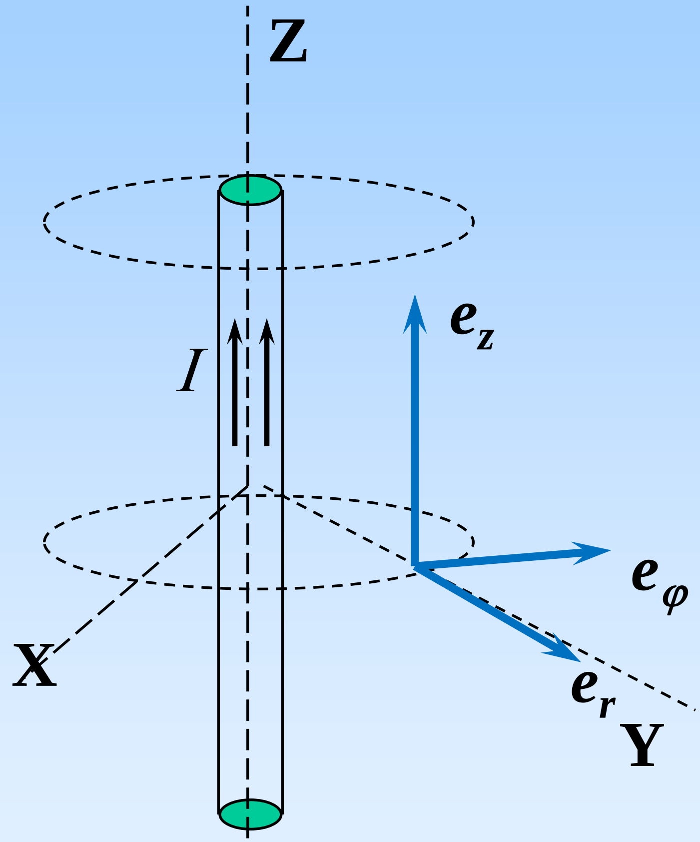
Question:

Calculate ***B***-field in arbitrary points around the wire

B-field of a long wire

- Analysis and symmetry
- Approach to solution
- Calculations
- Conclusions
- Appendix: angular integration

Analysis and Symmetry



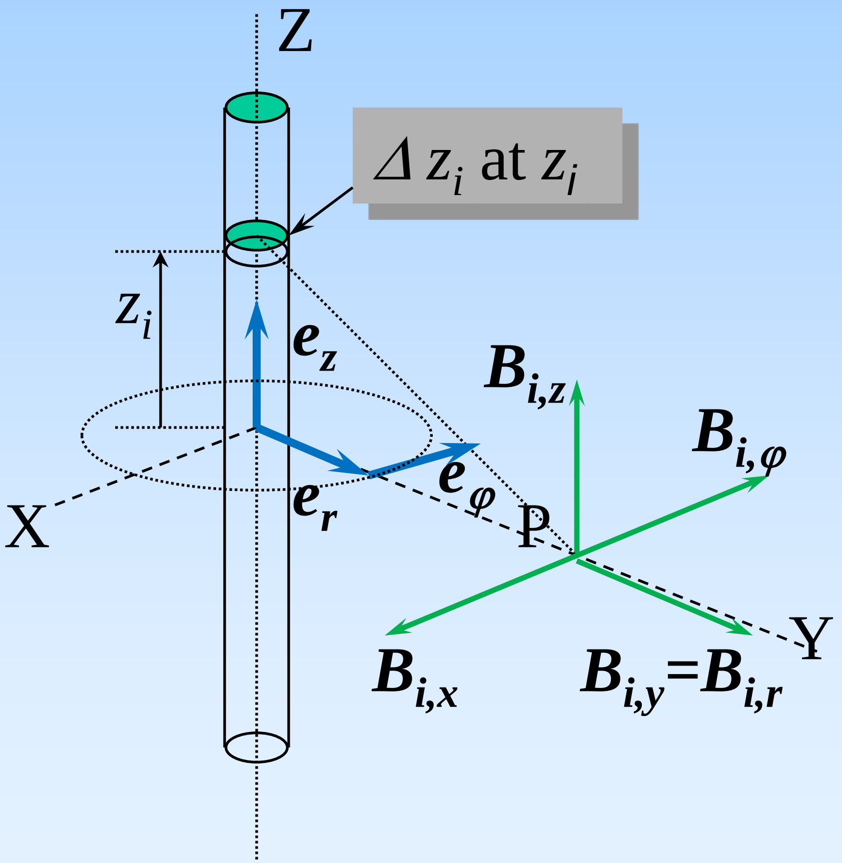
- Assume: thin wire
- Current: I [A]
- Coordinate axes:

  Z-axis // wire
- Symmetry: cylinder
- Cylinder coordinates:

  r, z, φ

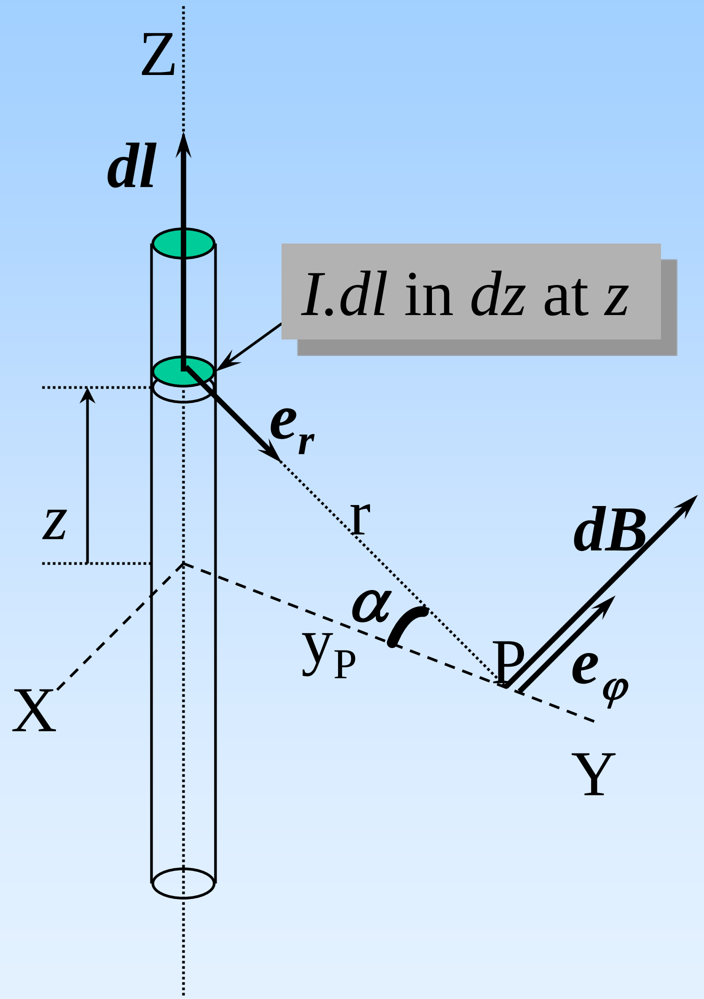
  all perpendicular !!

Analysis, field build-up



1. XYZ-axes
2. Point P on Y-axis
3. all $(I.\Delta z_i)$'s at z_i contribute $\boldsymbol{B_i}$ to $\boldsymbol{B}$ in P
4. $\boldsymbol{B_{i,x}}$, $\boldsymbol{B_{i,y}}$, $\boldsymbol{B_{i,z}}$
5. $\boldsymbol{e_{i,r}}$, $\boldsymbol{e_{i,z}}$, $\boldsymbol{e_{i,\varphi}}$
6. $\boldsymbol{B_{i,r}}$, $\boldsymbol{B_{i,z}}$, $\boldsymbol{B_{i,\varphi}}$

Approach to solution



current element *I.dl*

Biot & Savart :

$$\boldsymbol{dB} = \frac{\mu_0}{4\pi}\frac{I.\boldsymbol{dl} \times \boldsymbol{e_r}}{r^2}$$

dB in plane ⊥ wire Z-axis

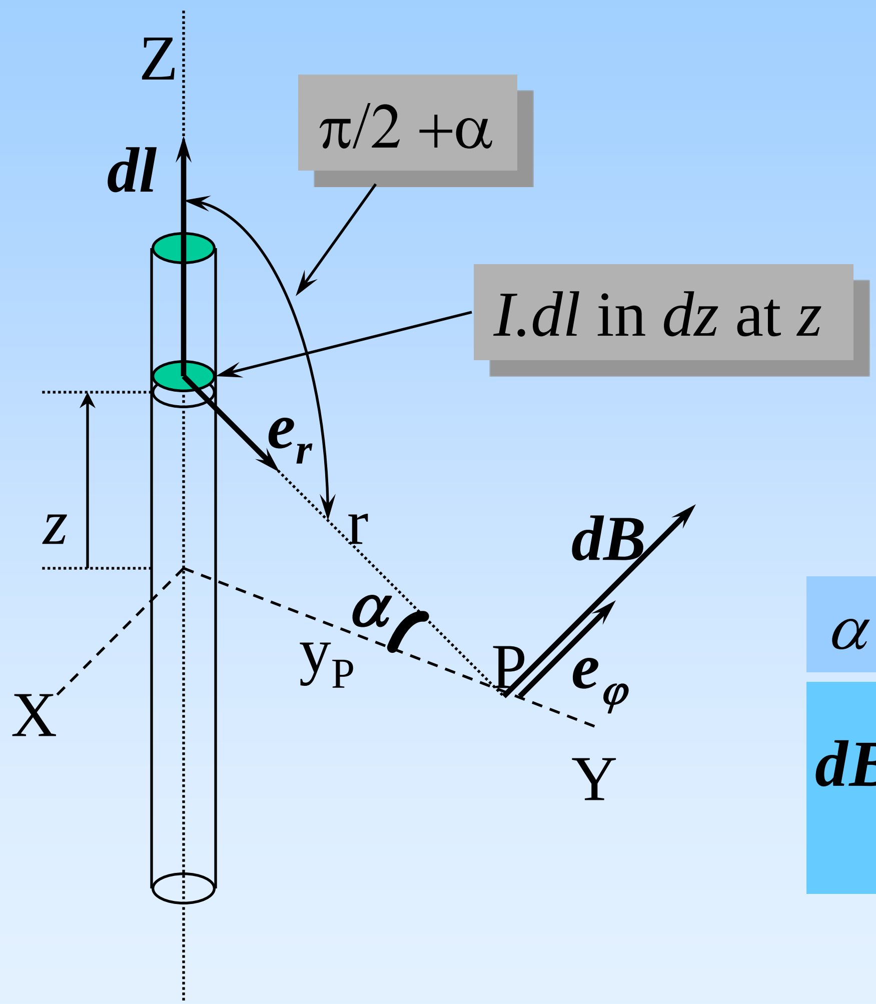
note: *r* and vector $\boldsymbol{e_r}$ ***!!***

dl x $\boldsymbol{e_r}$ => $\boldsymbol{\varphi}$ -comp. only !!

(valid for all *dl* in *l;*

→ **dB** in XY-plane)

Calculations



$$\boldsymbol{dB} = \frac{\mu_0}{4\pi}\frac{I.\boldsymbol{dl}\times\boldsymbol{e_r}}{r^2}$$

Calc. vector product, then *r*, both as *f(z)*, and insert

$$\boldsymbol{dl}\times\boldsymbol{e_r} = dz.\sin(\tfrac{1}{2}\pi+\alpha)\,\boldsymbol{e}_\varphi$$

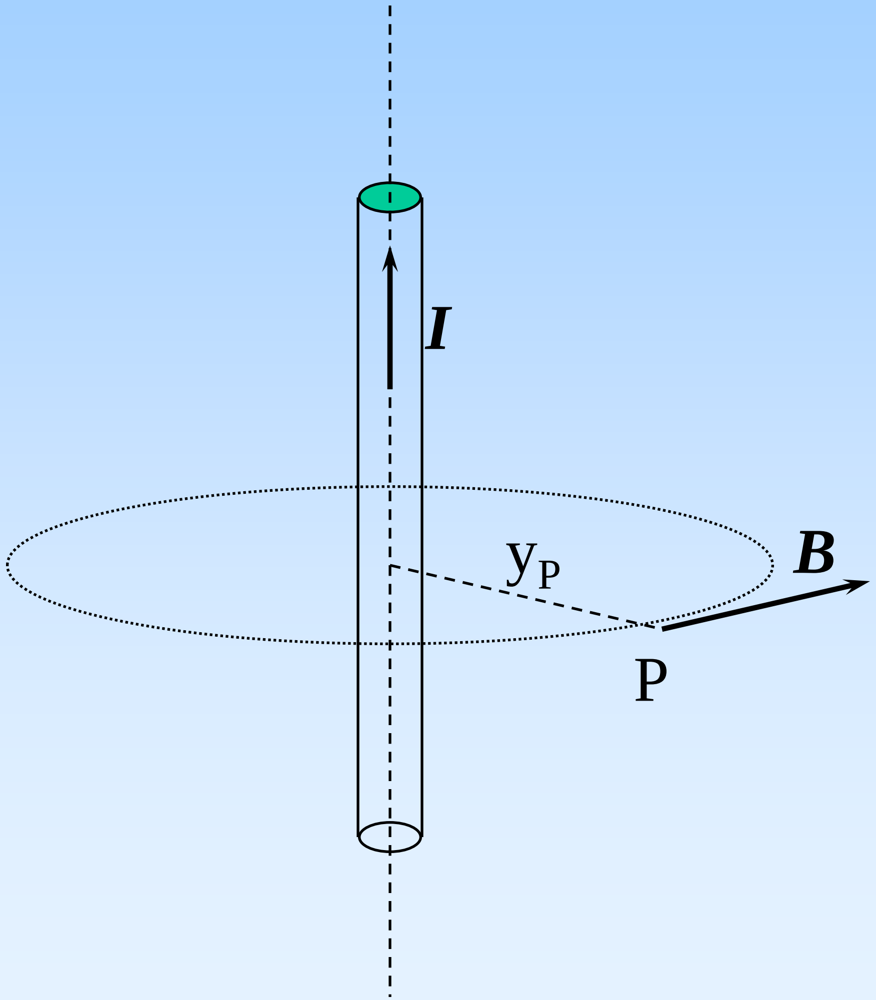
$$\ldots = dz.\cos\alpha\,\boldsymbol{e}_\varphi$$

α , r and $\boldsymbol{e_r}$ are *f(z)*

$$\boldsymbol{dB} = \frac{\mu_0.I.dz}{4\pi\left(z^2+y_P{}^2\right)}\frac{y_P}{\sqrt{z^2+y_P{}^2}}\,\boldsymbol{e}_\varphi$$

$$\boldsymbol{B} = \int_{-\infty}^{+\infty}\boldsymbol{dB} = \frac{\mu_0.I}{2\pi\,y_P}\,\boldsymbol{e}_\varphi$$

Conclusions



$$\boldsymbol{B} = \frac{\mu_0 I}{2\pi\ y_P} \boldsymbol{e}_\varphi$$

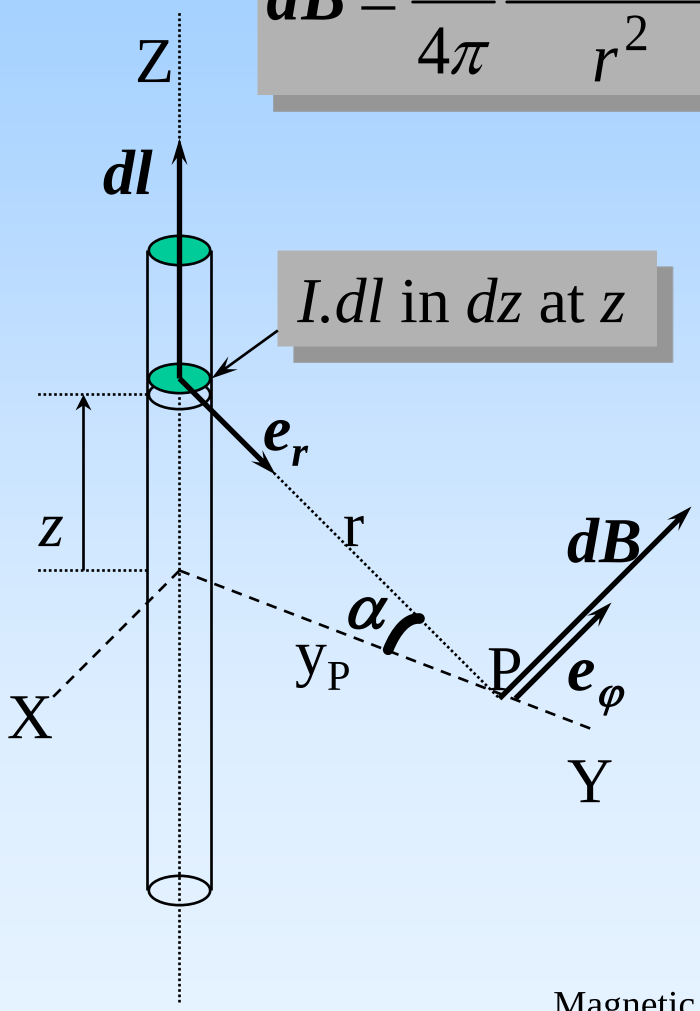
$|\boldsymbol{B}| \sim 1/y_P$

cylinder symmetry

B independent of *z* and *x*

Appendix: angular integration (1)

$$\boldsymbol{dB} = \frac{\mu_0}{4\pi}\frac{I.\boldsymbol{dl}\times\boldsymbol{e_r}}{r^2}$$



Calc. vector product, and *r*, both as *f(α)*, and insert in dB_φ

$\boldsymbol{dl} \times \boldsymbol{e_r} = dz\ .\ \sin(\tfrac{1}{2}\pi+\alpha)\ .\ \boldsymbol{e_\varphi}$

$= dz\ .\ \cos\alpha\ .\ \boldsymbol{e_\varphi}$

z, *dz*, *r* and $\boldsymbol{e_r}$ are *f(α)*

$z = y_P \tan\alpha$

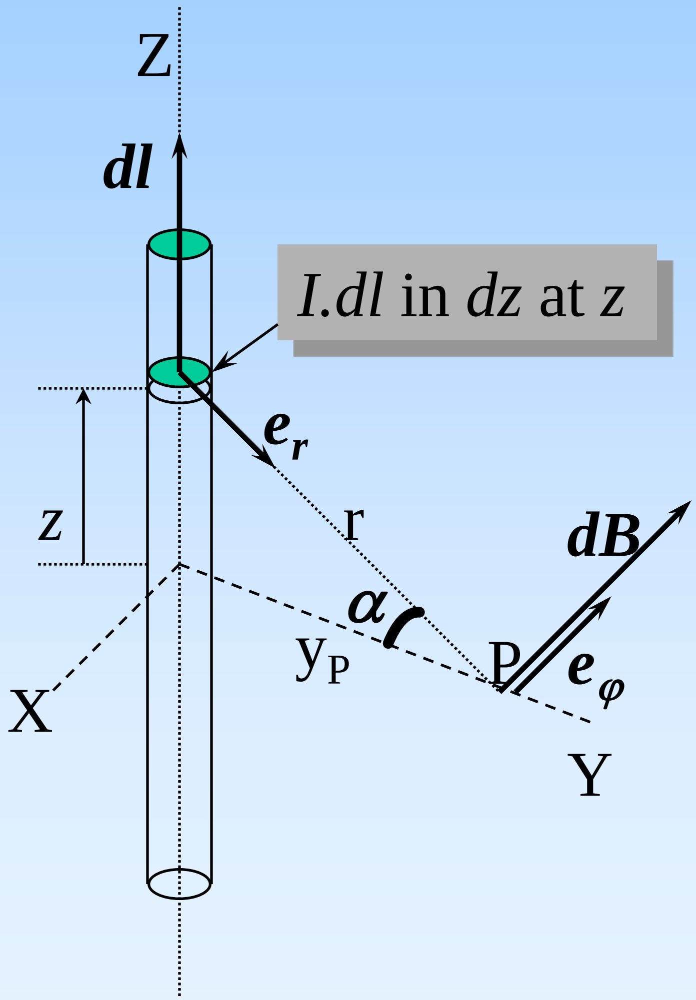
$r = y_P / \cos\alpha$

$$dz = \frac{y_P\, d\alpha}{\cos^2\alpha}$$

$$dB_\varphi = \frac{\mu_0 I\,(y_P/\cos^2\alpha)\,d\alpha}{4\pi(y_P^{\,2}/\cos^2\alpha)}\cos\alpha$$

$$dB_\varphi = \frac{\mu_0\, I}{4\pi\, y_P}\cos\alpha\, d\alpha$$

Appendix: angular integration (2)



$$dB_\varphi = \frac{\mu_0 I}{4\pi\, y_P} \cos\alpha\, d\alpha$$

integration of cos α over α

from -π/2 to π/2 :

$$\boldsymbol{B} = \frac{\mu_0 I}{2\pi\, y_P} \boldsymbol{e}_\varphi$$

the end