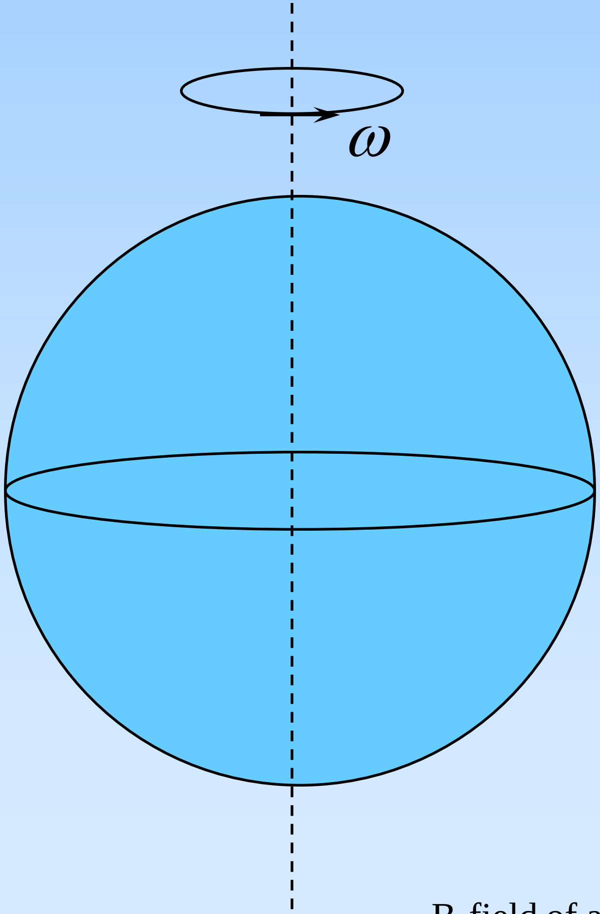
# Magnetic Field of a Rotating Homogeneously Charged Sphere

# Presentations:

- Electromagnetism: History
- Electromagnetism: Electr. topics
- Electromagnetism: Magn. topics
- Electromagnetism: Waves topics
- Capacitor filling (complete)
- Capacitor filling (partial)
- Divergence Theorem
- E-field of a thin long charged wire
- E-field of a charged disk
- E-field of a dipole
- E-field of a line of dipoles
- E-field of a charged sphere
- E-field of a polarized object
- E-field: field energy
- Electromagnetism: integrations
- Electromagnetism: integration elements
- Gauss' Law for a cylindrical charge
- Gauss' Law for a charged plane
- Laplace's and Poisson's Law
- B-field of a thin long wire carrying a current
- B-field of a conducting charged sphere
- B-field of a homogeneously charged sphere

# ***B***-field of a rotating homogeneously charged sphere



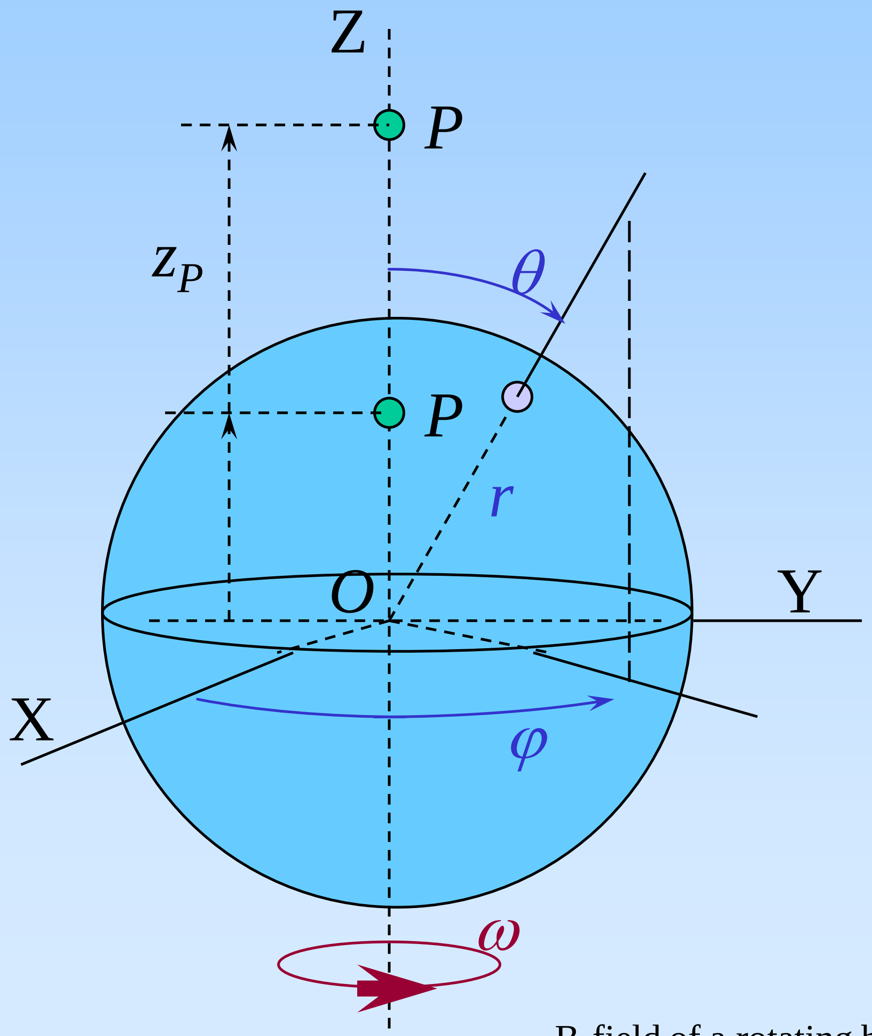

<u>Available:</u>

A charged sphere (charge $Q$, radius $R$), rotating with $\omega$ rad/sec

<u>Question:</u>

Calculate ***B***-field in arbitrary points on the axis of rotation inside and outside the sphere

# Analysis and Symmetry (1)



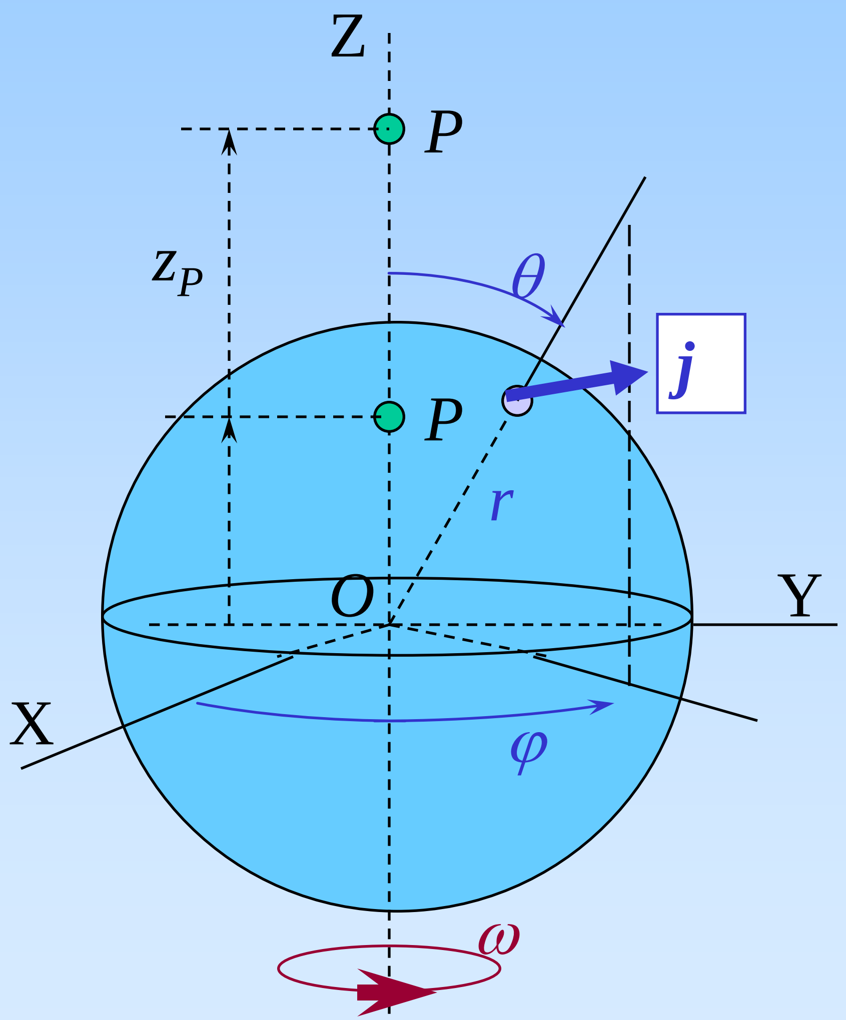

Calculate ***B***-field in point *P* inside or outside the sphere

Assume Z-axis through *O* and *P*.

Coordinate systems:

- X,Y, Z
- *r*, $\theta$, $\varphi$

# Analysis and Symmetry (2)




Homogeneously charged sphere ,

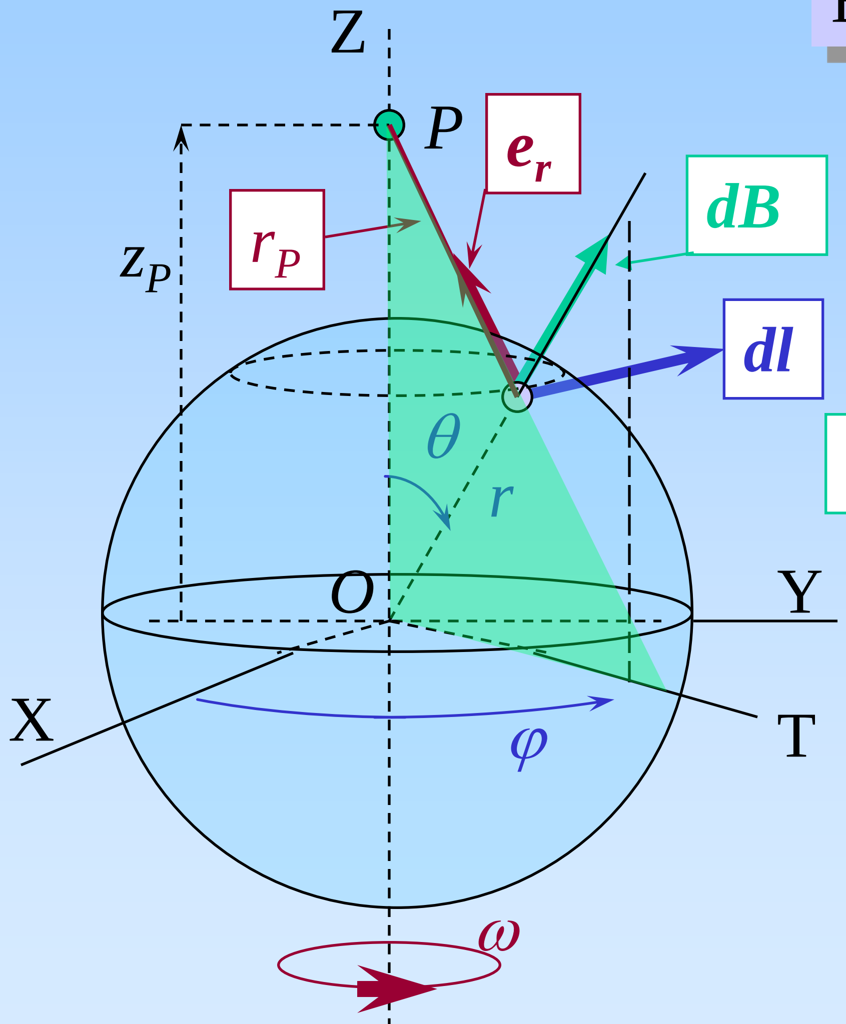
volume charge density:

$$\rho = Q/(4\pi R^3/3) \quad [\mathrm{C/m^3}]$$

Rotating charges will establish a “volume current”

Volume current density ***j*** $[\mathrm{A/m^2}]$ will be a function of $(r,\theta)$

# Analysis and Symmetry (3)

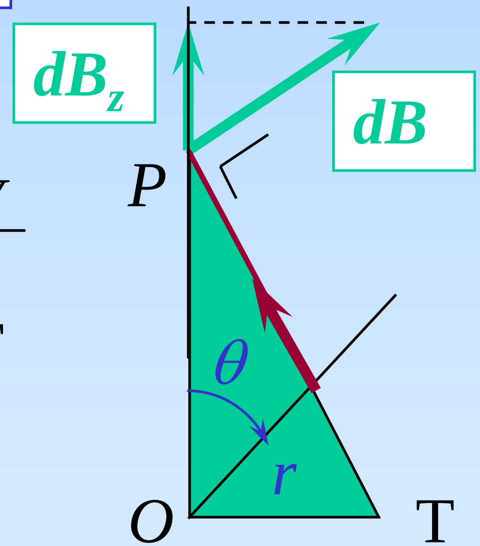



Biot & Savart :

$$dB = \frac{\mu_0}{4\pi} \frac{I . \boldsymbol{dl} \times \boldsymbol{e_r}}{r_P^{\;2}}$$

***dB*, *dl*** and ***e***$_{\boldsymbol{r}}$ mutual. perpendic!

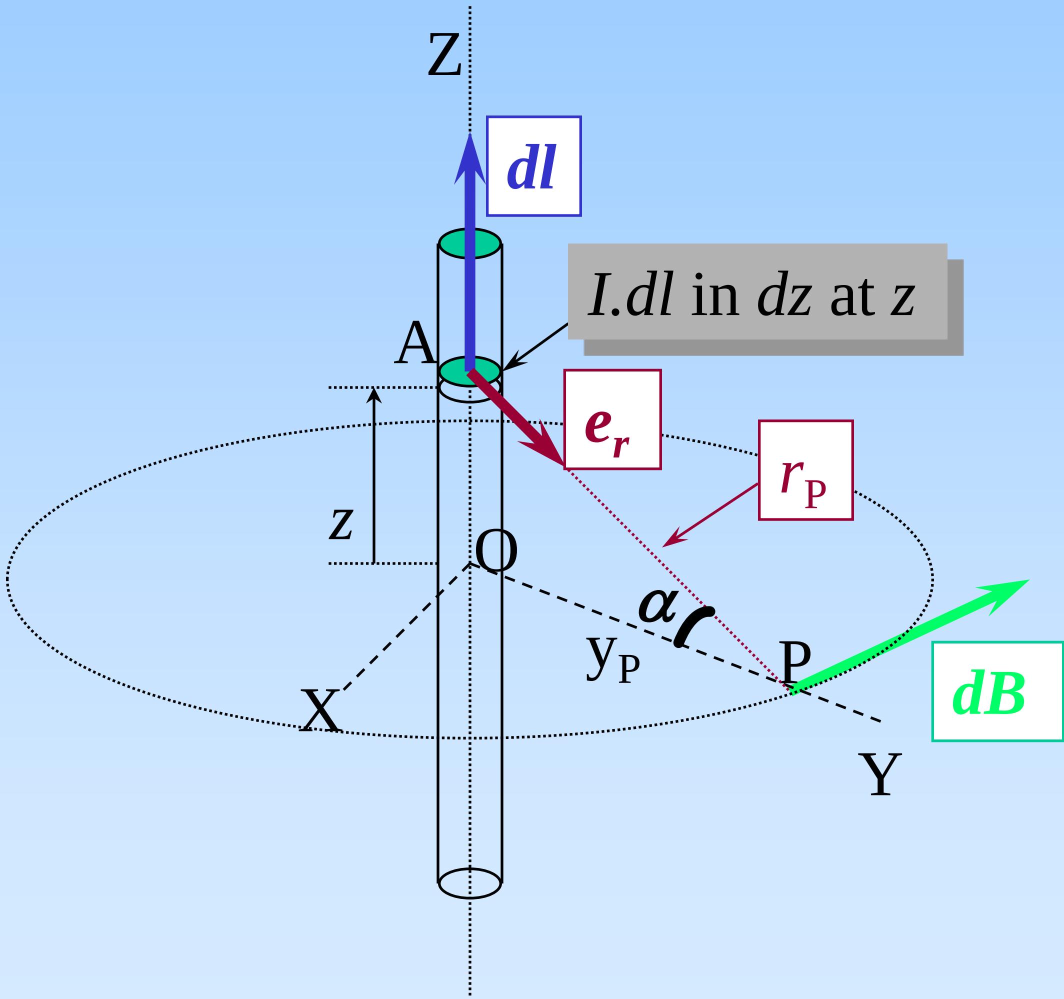
Direction of contributions *dB*:




Cylinder-symmetry around Z-axis:

Z-components only !!

# Intermezzo (1): a long wire




Biot & Savart :

$$\boldsymbol{dB} = \frac{\mu_0}{4\pi} \frac{I.\boldsymbol{dl} \times \boldsymbol{e_r}}{r_P^{\;2}}$$

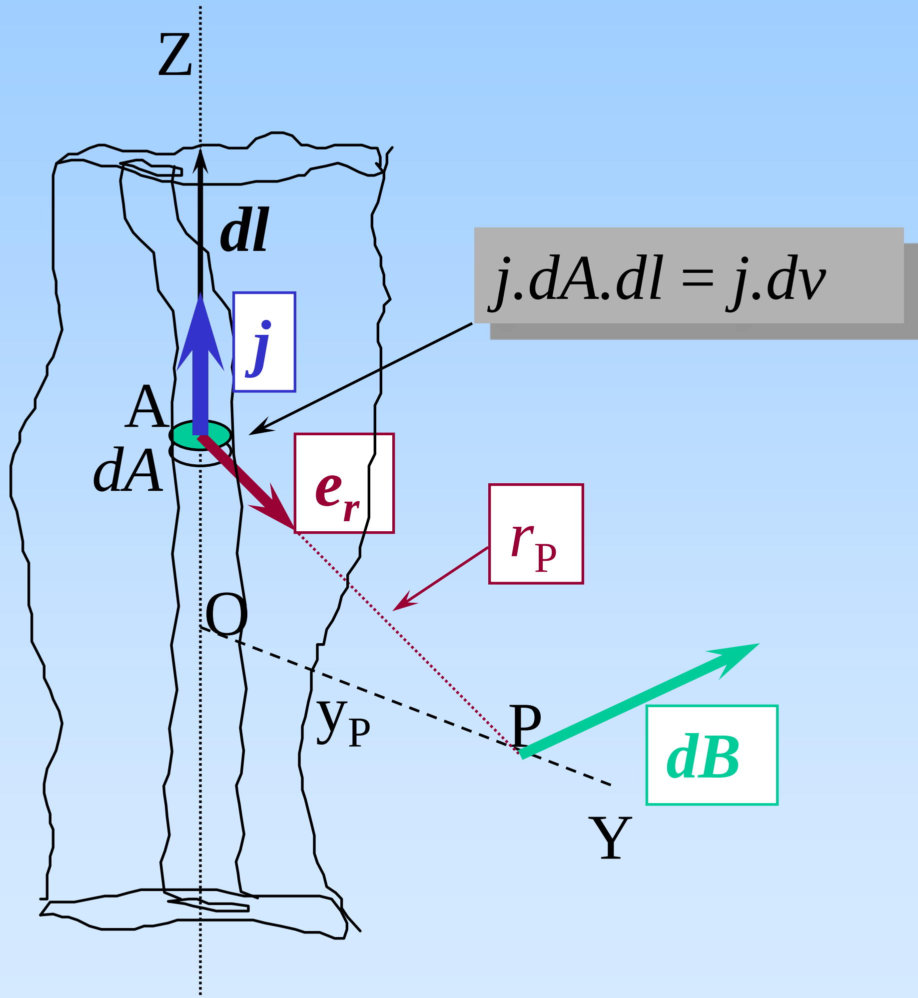
note:

$r$ and vector $\boldsymbol{e_r}$ !!

$\boldsymbol{dB} \perp \boldsymbol{dl}$ and $\boldsymbol{e_r}$

$\boldsymbol{dB} \perp \Delta$ AOP

# Intermezzo (2): a volume current




Biot & Savart :

$$\boldsymbol{dB} = \frac{\mu_0}{4\pi} \frac{\boldsymbol{j} \times \boldsymbol{e_r}}{r_P^{\;2}} dv$$

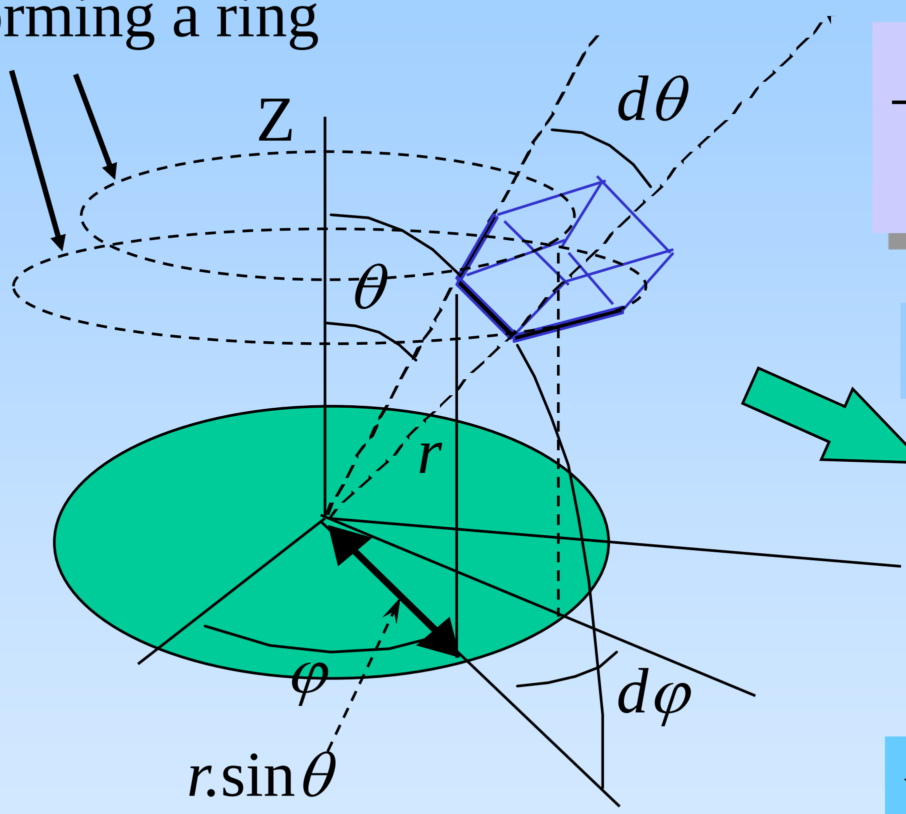
*j*: current density [A/m$^2$]

$\boldsymbol{dB} \perp \boldsymbol{dl}$ and $\boldsymbol{e_r}$

$\boldsymbol{dB} \perp \Delta$ AOP

# Intermezzo: volume current (3)

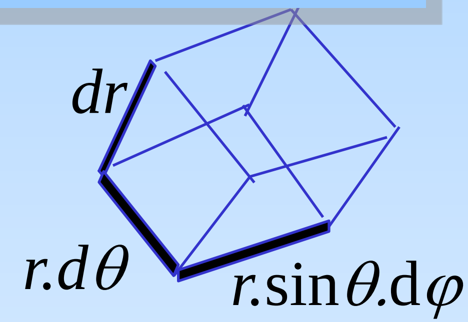
Two circles at $r$
forming a ring




Homogeneously charged sphere,

volume density: $\rho = Q/(4\pi R^3/3)$

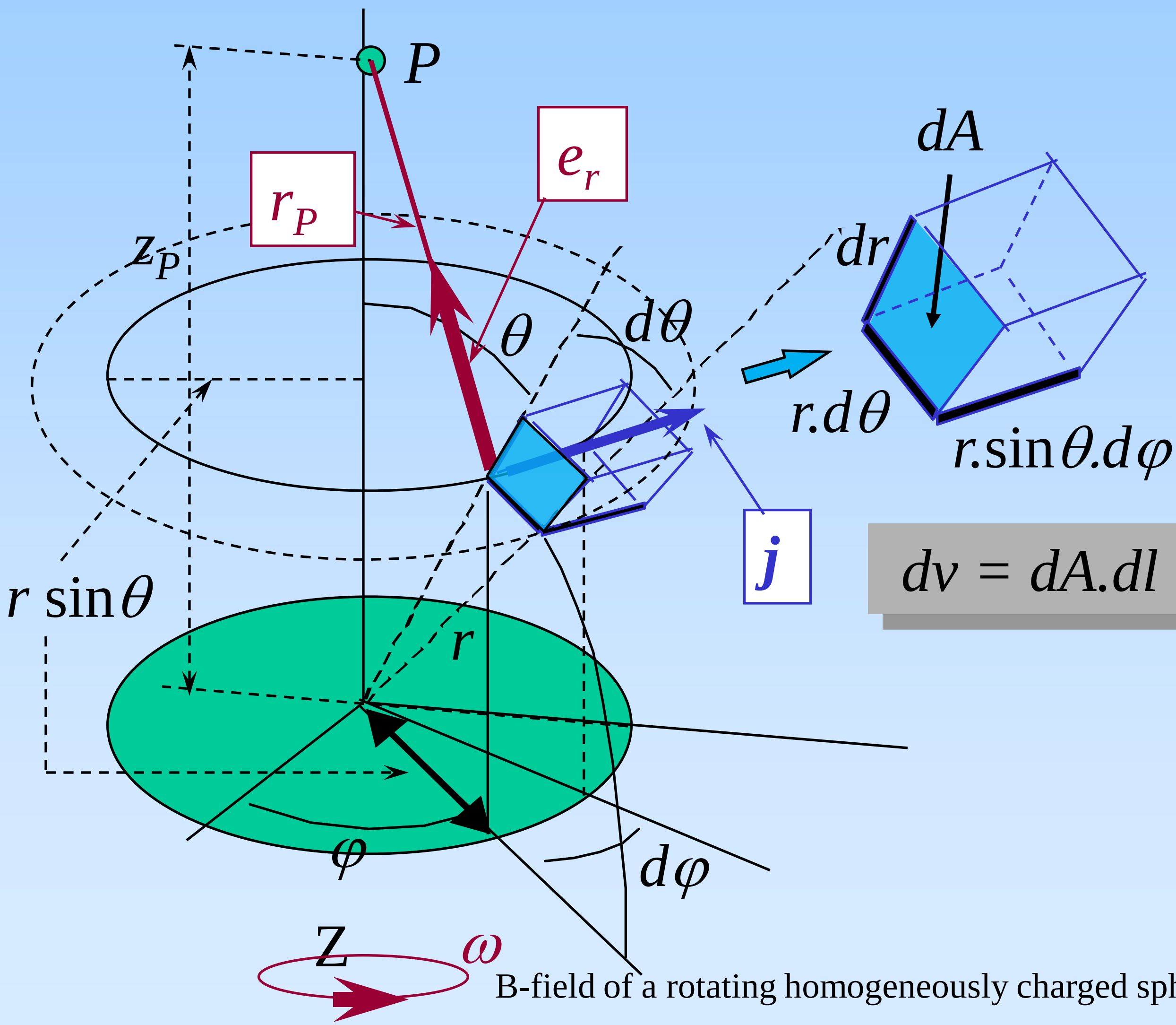
volume element:




volume element:

$dv = (dr).(r.d\theta).(r.\sin\theta.\ d\varphi)$

# Homogeneously charged sphere (1)




$$\boldsymbol{dB} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{j}\times\boldsymbol{e_r}}{r_P{}^2}dv$$

Charge $\rho.dv$ in $dv$ will rotate with $\omega$

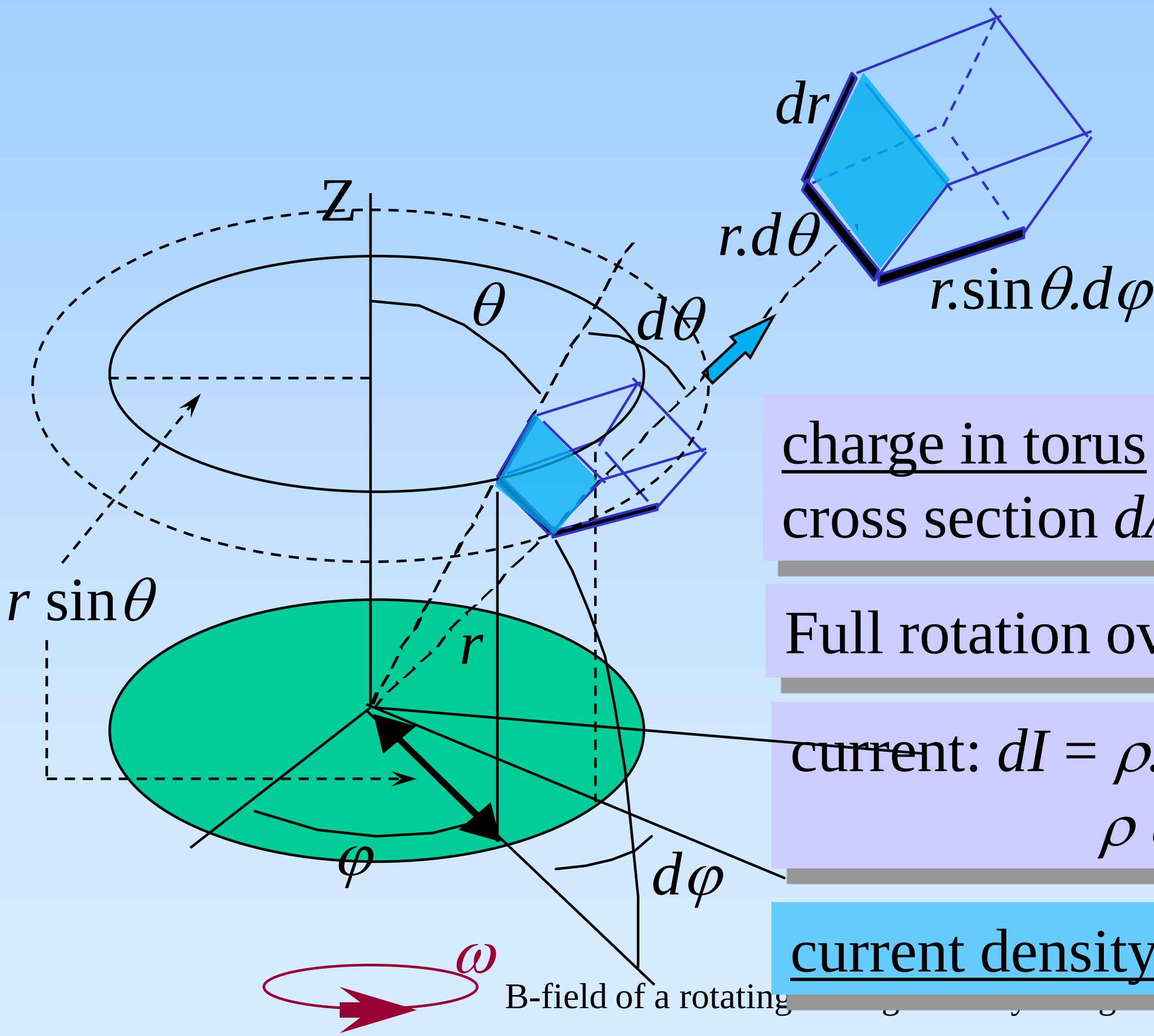
$dv = dA.dl$

$dl = r.\sin\theta.d\varphi$
$dA = dr.rd\theta$

Needed:

- $\boldsymbol{j}$ , $\boldsymbol{e_r}$ , $r_P$

# Homogeneously charged sphere (2)




$dv = dA.dl$

$dl = r.\sin\theta.d\varphi$ ;
$dA = dr.rd\theta$

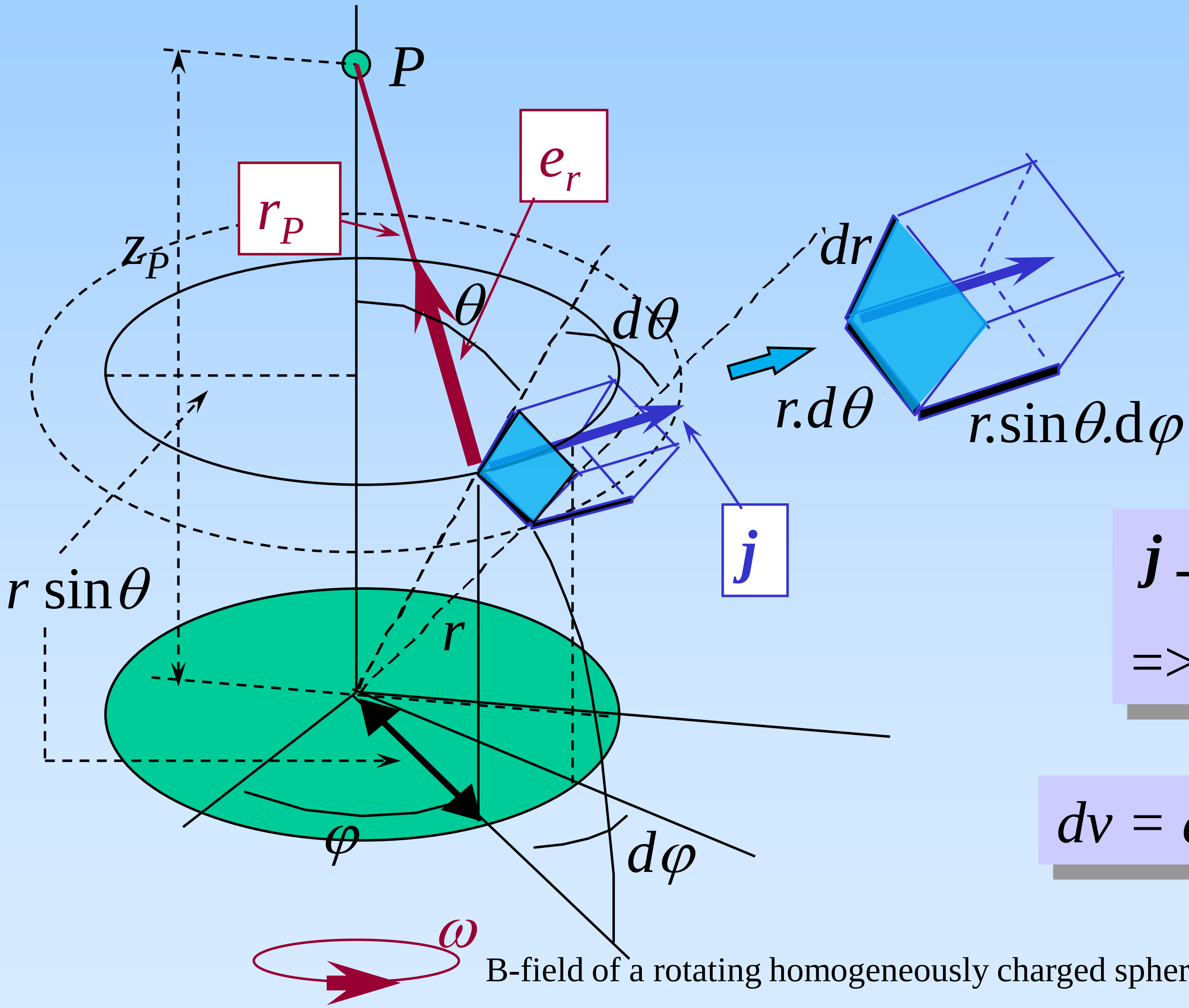
<u>charge in torus</u> with radius $r.\sin\theta$ and cross section $dA$ is: $\rho\, 2\pi r.\sin\theta\, .\, dA$

Full rotation over $2\pi r$ in $2\pi/\omega$ s.

current: $dI = \rho.2\pi r.\sin\theta.dA\, /\, (2\pi/\omega) =$
$\rho\,\omega\, r\,\sin\theta\, .dA$

<u>current density:</u> $j = \rho\omega r\,\sin\theta$ [A/m²]

# Homogeneously charged sphere (3)




$$dB = \frac{\mu_0}{4\pi} \frac{\boldsymbol{j} \times \boldsymbol{e_r}}{r_P^{\ 2}} dv$$

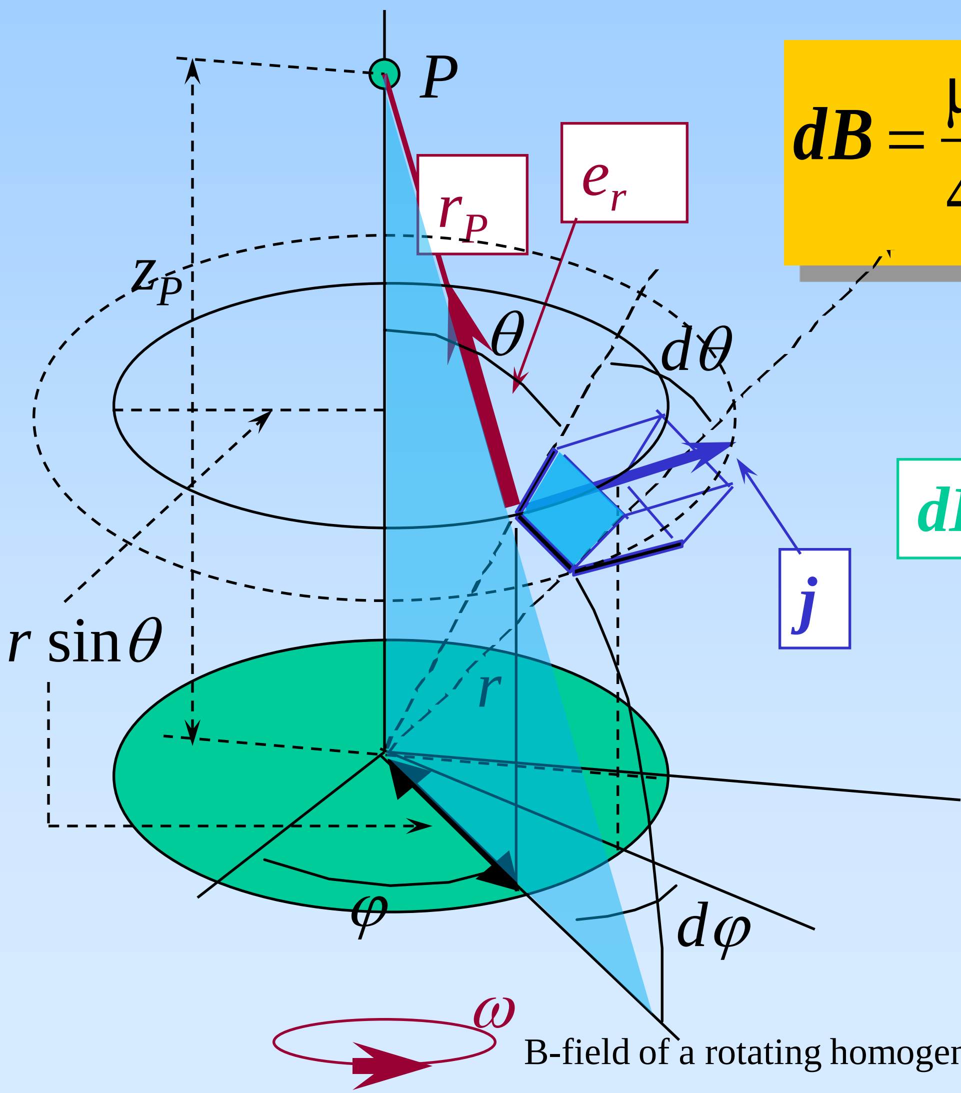
$$j = \rho\, \omega\, r \sin\theta$$

$\boldsymbol{j} \perp \boldsymbol{e_r}$ :

=> $|\boldsymbol{j} \times \boldsymbol{e_r}| = j.e_r = j$

$$dv = dr.\ \ r.d\theta.\ \ r.\sin\theta.\ d\varphi$$

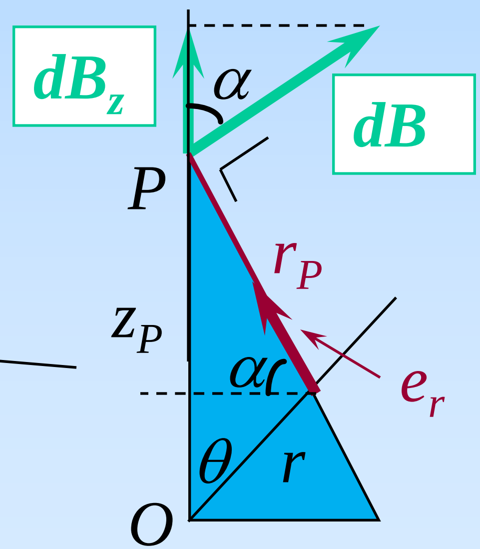
# Homogeneously charged sphere (4)




$$dB = \frac{\mu_0}{4\pi} \frac{j \times e_r}{r_P^{\;2}} dv$$

$$j = \rho \omega r \sin\theta$$

$$dv = dr.\ \ r.d\theta.\ \ r.\sin\theta.\,d\varphi$$

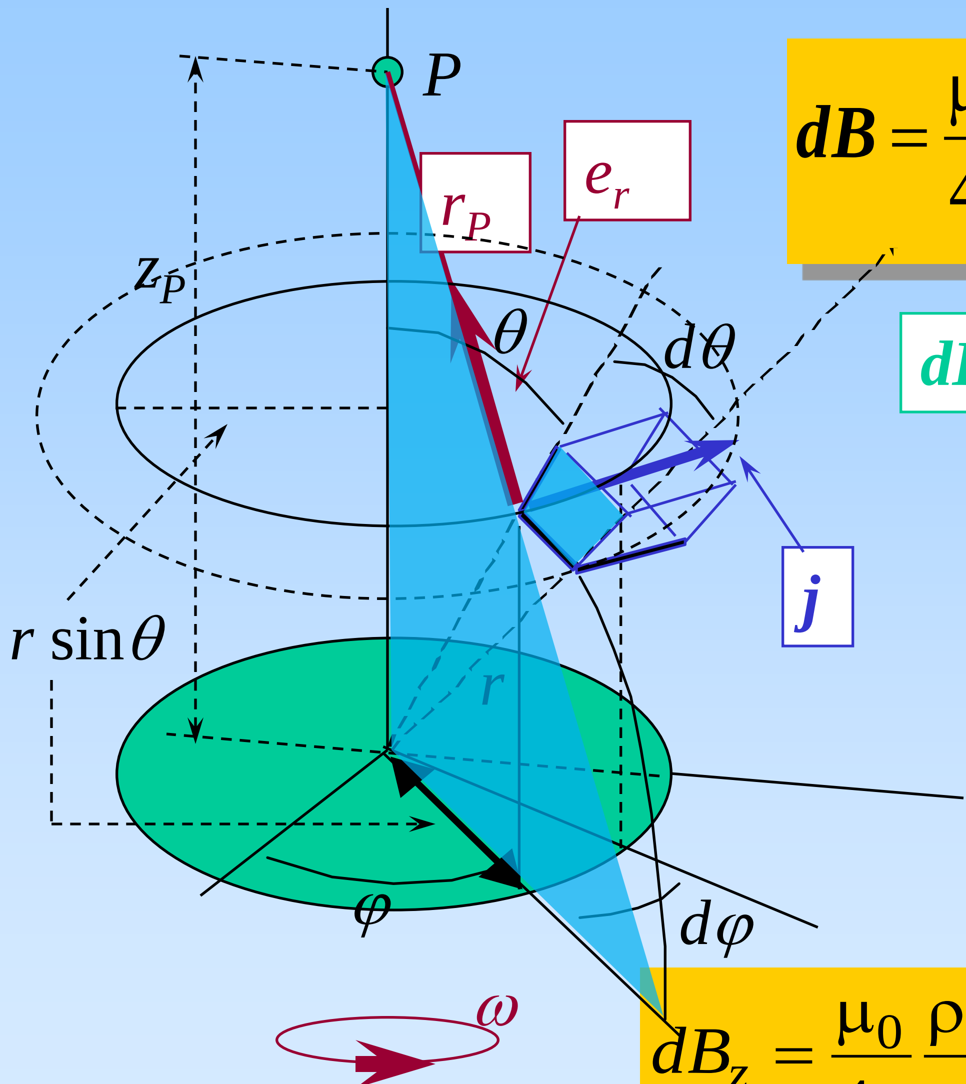



Cylinder-symmetry:

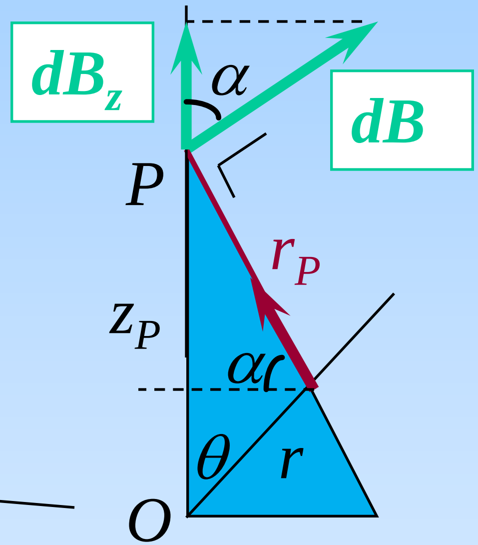
Z-components only !!

$$\cos\alpha = \frac{r \sin\theta}{r_P}$$

# Homogeneously charged sphere (5)




$$dB = \frac{\mu_0}{4\pi}\frac{j \times e_r}{r_P{}^2}dv$$




$$j = \rho\omega r\sin\theta$$

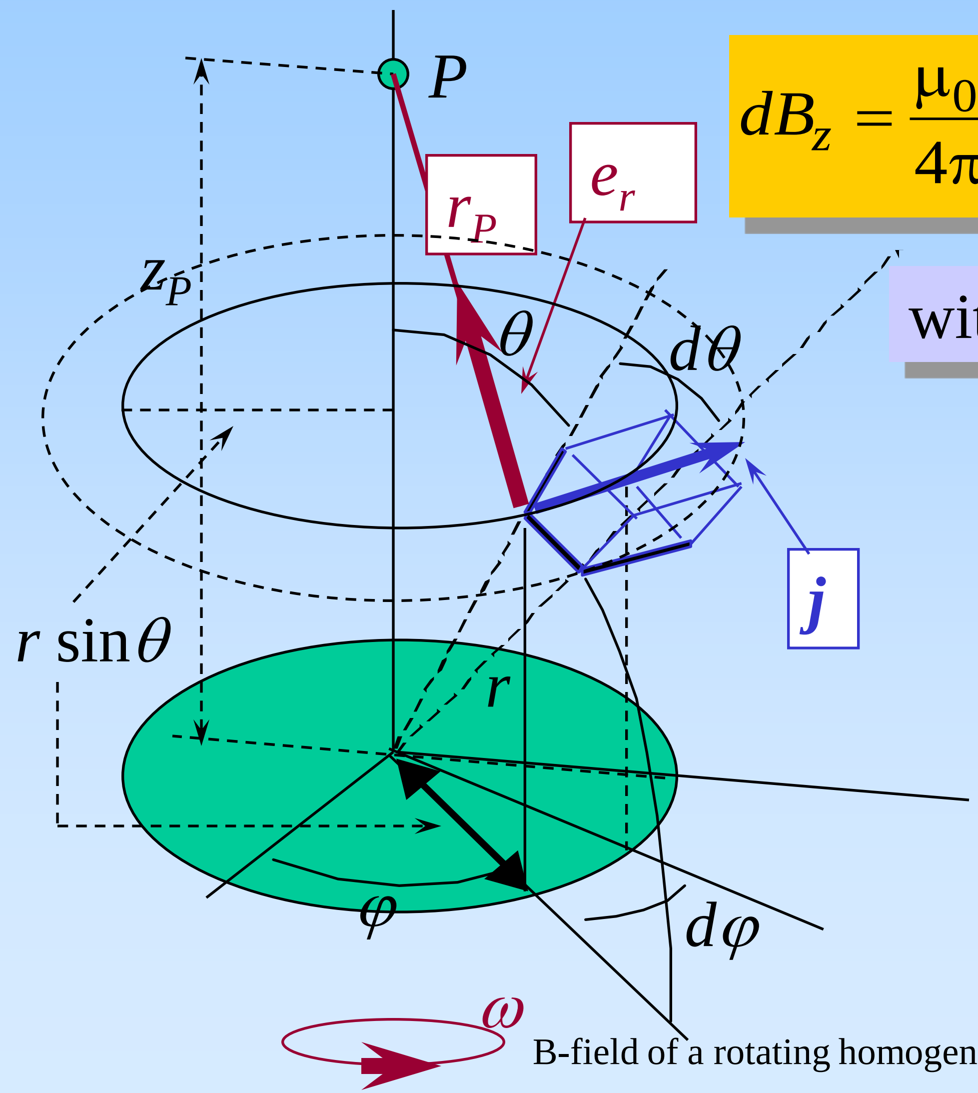
$$dv = dr.\ r.d\theta.\ r.\sin\theta.\ d\varphi$$

$$\cos\alpha = \frac{r\sin\theta}{r_P}$$

$$r_P{}^2 = (r.\sin\theta)^2 + (z_P - r.\cos\theta)^2$$

$$dB_z = \frac{\mu_0}{4\pi}\frac{\rho\omega r\sin\theta}{r_P{}^2}dr.r.d\theta.r.\sin\theta d\varphi\frac{r\sin\theta}{r_P}$$

# Homogeneously charged sphere (6)




$$dB_z = \frac{\mu_0}{4\pi}\frac{\rho\omega r\sin\theta}{{r_P}^2}dr.r.d\theta.r.\sin\theta d\varphi\frac{r\sin\theta}{r_P}$$

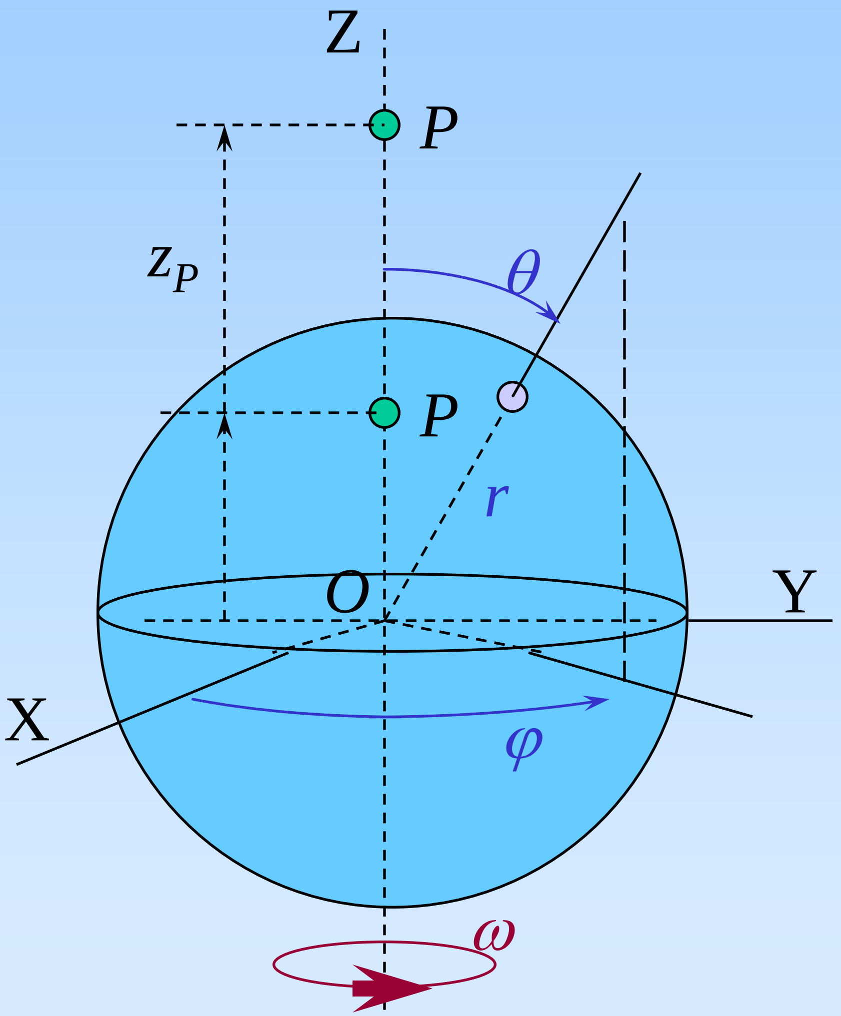
with $r_P{}^2 = (r.\sin\theta)^2 + (z_P - r.\cos\theta)^2$

$$dB_z = \frac{\mu_0}{4\pi}\frac{\rho\omega r^4\sin^3\theta}{{r_P}^3}dr.d\theta.d\varphi$$

Integration:

$0<r<R$ ; $0<\theta<\pi$ ; $0<\varphi<2\pi$

# Homogeneously charged sphere (7)




B-field of a rotating homogeneously charged sphere

result : $\boldsymbol{B} = \dfrac{2\mu_0 \rho \omega R^5}{15.z_p{}^3}\boldsymbol{e_z}$

this result holds for $z_P > R$ ;

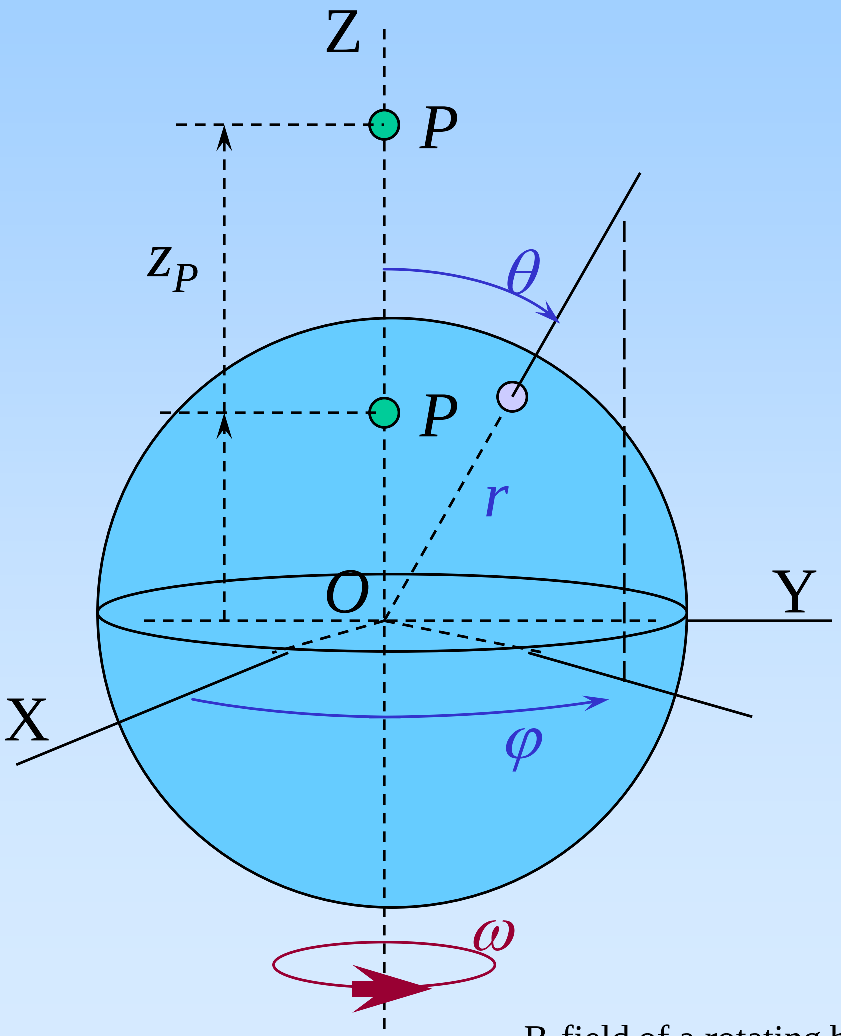
for -R< $z_P$ < $R$ the result is:

$$\boldsymbol{B} = \frac{\mu_0 \rho \omega R^3}{15.z_p{}^3}\left(5R^2 - 3z_P{}^2\right)\boldsymbol{e_z}$$

and for $z_P < -R$:

$$\boldsymbol{B} = \frac{2\mu_0 \rho \omega R^5}{-15.z_p{}^3}\boldsymbol{e_z}$$

# Homogeneously charged sphere (8)




result for $|z_P| > R$ :

$$\boldsymbol{B} = \frac{2\mu_0 \rho \omega R^5}{15.\left|z_p{}^3\right|} \boldsymbol{e_z}$$

result for $|z_P| < R$ :

$$\boldsymbol{B} = \frac{\mu_0 \rho \omega R^3}{15.z_p{}^3} \left(5R^2 - 3z_P{}^2\right) \boldsymbol{e_z}$$

***B*** points along +$\boldsymbol{e_z}$ for all points everywhere on Z-axis !!

# Homogeneously charged sphere (9)

With volume density: $\rho = Q/(4\pi R^3/3)$ :

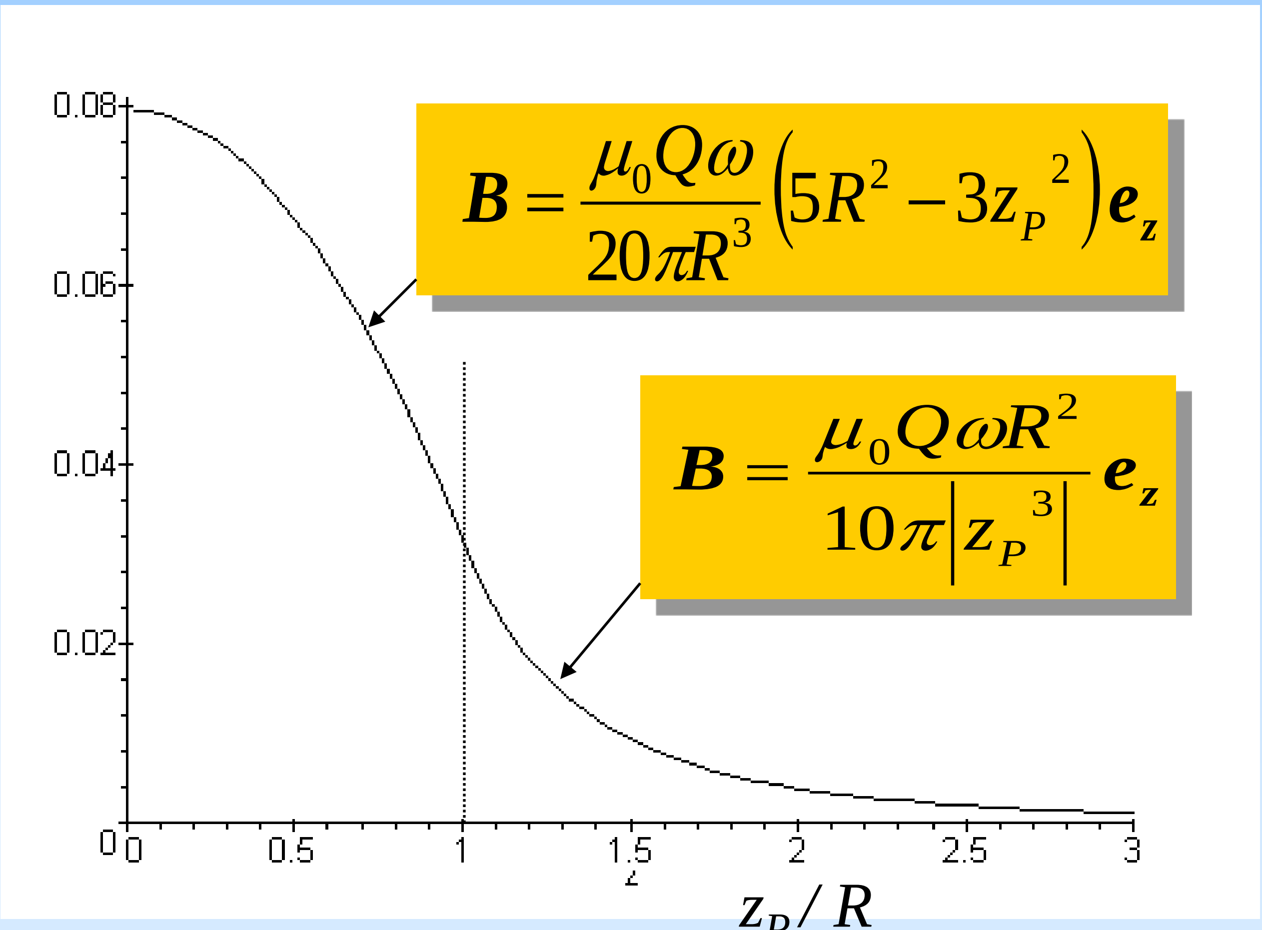
result for
$|z_P| > R$ :

$$\boldsymbol{B} = \frac{2\mu_0 \rho \omega R^5}{15 . \left| z_p{}^3 \right|} \boldsymbol{e_z} = \frac{\mu_0 Q \omega R^2}{10\pi \left| z_P{}^3 \right|} \boldsymbol{e_z}$$

result for
$|z_P| < R$ :

$$\boldsymbol{B} = \frac{\mu_0 \rho \omega}{15} \left( 5R^2 - 3z_P{}^2 \right) \boldsymbol{e_z} = \frac{\mu_0 Q \omega}{20\pi R^3} \left( 5R^2 - 3z_P{}^2 \right) \boldsymbol{e_z}$$

# Homogeneously charged sphere (10)




$$\boldsymbol{B} = \frac{\mu_0 Q \omega}{20 \pi R^3}\left(5R^2 - 3{z_P}^2\right)\boldsymbol{e_z}$$

$$\boldsymbol{B} = \frac{\mu_0 Q \omega R^2}{10 \pi \left|{z_P}^3\right|}\boldsymbol{e_z}$$

Plot of $\boldsymbol{B}$ for:

$Q = 1$

$\mu_0 = 1$

$\omega = 1$

(in SI-units)

# Conclusions (1)

## Homogeneously charged sphere

$|z_P| > R$

$$\boldsymbol{B} = \frac{\mu_0 Q \omega R^2}{10\pi \left| z_P{}^3 \right|} \boldsymbol{e_z}$$

$|z_P| < R$

$$\boldsymbol{B} = \frac{\mu_0 Q \omega}{20\pi R^3} \left( 5R^2 - 3z_P{}^2 \right) \boldsymbol{e_z}$$

## Conducting sphere (see other presentation)

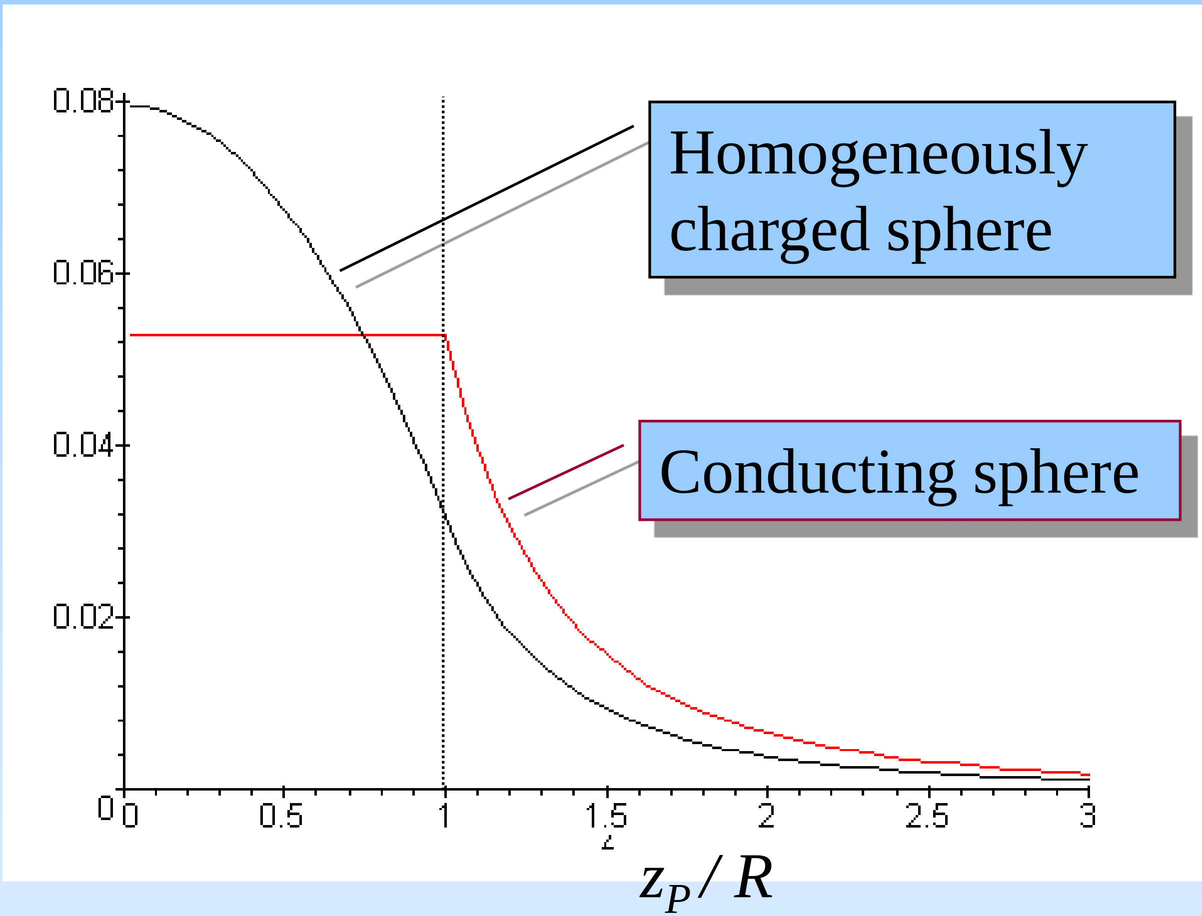
$|z_P| > R$

$$\boldsymbol{B} = \frac{\mu_0 Q \omega R^2}{6\pi \left| z_p{}^3 \right|} \boldsymbol{e_z}$$

$|z_P| < R$

$$\boldsymbol{B} = \frac{\mu_0 Q \omega}{6\pi R} \boldsymbol{e_z}$$

# Conclusions (2)




Plot of ***B*** for:

$Q = 1$

$\mu_0 = 1$

$\omega = 1$

(in SI-units)

*The end!!*